

Resonant (soft) x-ray scattering

SUCCESS-2014 : School on UV and X-ray spectroscopies of correlated electron systems

1-12 Sept. 2014

Peter Abbamonte



University of Illinois at Urbana-Champaign
abbamonte@mrl.illinois.edu
<http://users.mrl.illinois.edu/abbamonte/>

Resonant X-ray Scattering Demystified

Note Title

8/28/2011

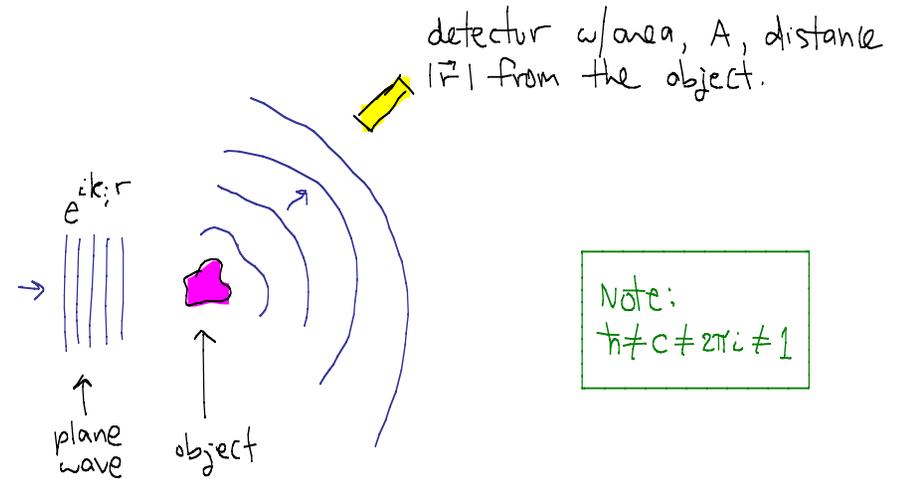
We will be talking about x-ray "diffraction", which is a specific type of scattering in which the photon frequency does not change. Before proceeding, one must first make two observations:

1. Experimenters rarely measure only diffraction. So-called "XRD" employs no energy analyzer, meaning one really is doing energy-integrated scattering. Under non-resonant conditions what one really measures is $\int S(\mathbf{q}, \omega) d\omega$, which is an equal-time correlation function. Such experiments must, then, work under conditions where the elastic part is dominant, such as near a Bragg reflection.

2. If the elastic part can be isolated, classical scattering theory is valid. Said another way, if there is no (coherent) interchange of energy between the radiation field and the system, the propagation of the field can be described by Maxwell's equations, all the quantum mechanics being hidden in the optical constants, χ , ϵ , n , σ , etc. This allows a separation of the classical scattering problem from the QM problem of computing χ .

P. Abbamonte, Les Houches, Sept. 2014

In "elastic" scattering we are concerned with the following situation:



The goal here is, by examining the scattering, to learn as much as possible about the object

Local Scattering

So far, basically all treatments of the scattering problem in the literature make the assumption of locality in the optical constants. Let's consider what this means.

Generally speaking, the dielectric susceptibility relates the polarization to the (total) electric field:

$$\vec{P}(\vec{r}, t) = \int d\vec{r}' dt' \chi_e(\vec{r}, \vec{r}', t-t') \vec{E}(\vec{r}', t')$$

[Note: in general this is a matrix product]

In other words, χ is a Green's function that describes how \vec{P} arises from \vec{E} . The resulting charge density

$$\rho_b(\vec{r}, t) = -\nabla \cdot \vec{P}$$

The approximation normally made in scattering is

$$\chi_e(\vec{r}, \vec{r}', t-t') = \chi_e(\vec{r}, t-t') \delta^3(\vec{r}-\vec{r}')$$

i.e.,
$$\vec{P}(\vec{r}, t) = \int dt' \chi_e(\vec{r}, t-t') \vec{E}(\vec{r}, t')$$

By the convolution theorem,

$$\vec{P}(\vec{r}, \omega) = \chi_e(\vec{r}, \omega) \vec{E}(\vec{r}, \omega)$$

In an anisotropic medium this is really a matrix product

$$P_i(\vec{r}, \omega) = \chi_{ij}^e(\vec{r}, \omega) E_j(\vec{r}, \omega)$$

In other words, everyone assumes that the medium is locally polarizable, i.e. that it can be described by a spatially-varying susceptibility, $\chi(\vec{r})$, dielectric constant, $\epsilon(\vec{r})$, etc.

Is approximation valid? It depends. For **nonresonant scattering** it is **rigorously correct**. For resonant scattering it is valid when the mean free path of the excitations that propagate \vec{P} is much less than the length scale on which one is probing the system, i.e. when

$$\lambda_{\text{MFP}} \ll \frac{2\pi}{g} \quad \text{or} \quad \lambda_{\text{MFP}} g \ll 2\pi$$

where g is the momentum transfer in the experiment. This condition is often satisfied—but not always.

If it fails, the scattering problem can still be treated classically, but requires deriving a nonlocal version of Maxwell's equations.

Classical, Rayleigh-Schrodinger Scattering

[Jackson, Classical Electrodynamics, 3rd Ed., Ch 10]

We will henceforth assume our system can be described by a spatially-varying dielectric susceptibility, $\chi(\vec{r}, \omega)$

$\nabla \cdot \vec{B} = 0$	$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$	$\vec{D} = \epsilon \vec{E}$
$\nabla \cdot \vec{E} = 0$ (no free charge)	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{H} = \frac{1}{\mu} \vec{B}$
	Faraday's law	$\epsilon(\vec{r}) = 1 + \chi_e(\vec{r})$

To get scattering one must solve the wave equation, which is normally acquired by taking the curl of Faraday's law. Leaving out some steps, we get

$$\nabla^2 \vec{D} - \frac{1}{c^2} \frac{\partial^2 \vec{D}}{\partial t^2} = -\nabla \times \nabla \times (\vec{D} - \epsilon_0 \vec{E}) + \epsilon_0 \frac{\partial}{\partial t} \nabla \times (\vec{B} - \mu_0 \vec{H})$$

$$= -\nabla \times \nabla \times \vec{P}(\vec{r}, t) + \frac{1}{c^2} \frac{\partial}{\partial t} \nabla \times \vec{M}(\vec{r}, t)$$

where $\vec{P} = \epsilon_0 \chi_e(\vec{r}) \vec{E}$ and $\vec{M} = \chi_m(\vec{r}) \vec{H}$

In a uniform medium (χ_e, χ_m const) the RHS vanishes and you get the classical wave equation whose solutions are plane waves.

Since there is a scatterer, however, the RHS acts like a source. To solve this, we need to use a Green's function. Take out the harmonic time dependence and we get the Helmholtz equation

$$(\nabla^2 + k^2) \vec{D} = -\nabla \times \nabla \times \vec{P} - \frac{i\omega}{c^2} \nabla \times \vec{M}$$

The Green's function for Helmholtz eq is $G = -\frac{1}{4\pi} \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$ so we can write down the soln.

$$\vec{D}(\vec{r}) = \vec{D}^{(i)} + \frac{1}{4\pi} \int d^3r' \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} (\nabla' \times \nabla' \times \vec{P}(\vec{r}') + \frac{i\omega}{c^2} \nabla' \times \vec{M}(\vec{r}'))$$

↑
Particular solution that meets the boundary

conditions as $\vec{r} \rightarrow \infty$, i.e., the incident wave.

This is now an integral equation, i.e., the integrand depends on the solution.

Still, it can be simplified somewhat. First, since \vec{P} and \vec{M} are nonzero only near the sample, and the detector is far away we can make the approximation $\vec{r} \rightarrow \vec{r}'$:

$$\vec{D} = \vec{D}^{(i)} + \vec{A}_{sc} \frac{e^{ikr}}{r}$$

A little integration by parts gives the scattering amplitude

$$\vec{A}_{sc} = \frac{k^2}{4\pi} \int d^3r' e^{-ik_f \cdot \vec{r}'} \left[\hat{n} \times \vec{P}(\vec{r}') \times \hat{n} - \frac{\epsilon_0 \omega}{k} \hat{n} \times \vec{M}(\vec{r}') \right]$$

where $\hat{n} = \vec{r}/r$ and $\vec{k}_f = k\hat{n}$. This is called far field diffraction or, sometimes, **Fraunhofer diffraction**. Guess, physically, the electric field @ the detector.

We now have to contend with the fact that we don't know \vec{P} and \vec{M} , which depend on the fields: To this end, people usually appeal to the **first Born approximation**, or the single-scattering approximation, which is based on the assumption that the scattering is "weak", i.e.

$$\vec{P}(\vec{r}) \approx \chi_e(\vec{r}) \vec{D}^{(i)}(\vec{r}) \quad \vec{M}(\vec{r}) \approx \chi_m(\vec{r}) \vec{H}^{(i)}(\vec{r})$$

$$\vec{D}^{(i)}(\vec{r}) = \hat{\epsilon}_i D_0 e^{i\vec{k}_i \cdot \vec{r}} \quad \hat{\epsilon}_i = \text{incident polarization}$$

$$\vec{H}^{(s)}(\vec{r}) = \sqrt{\frac{\mu_0}{\epsilon_0}} \hat{k}_i \times \vec{D}^{(i)} \quad \hat{\epsilon}_f = \text{scattered polarization}$$

This is effectively a truncation of the integral equation, and is called the **first Born approximation** in Rayleigh-Schrödinger perturbation theory.

Plug these in and do the cross products

$$\hat{\epsilon}_f^* \cdot \vec{A}_{sc}^{(i)} = D_0 \frac{k^2}{4\pi} \int d\vec{r} e^{i\vec{q} \cdot \vec{r}} \left[(\hat{\epsilon}_f^* \cdot \hat{\epsilon}_i) \chi_e(\vec{r}) + (\hat{k}_f \times \hat{\epsilon}_f^*) \cdot (\hat{k}_i \times \hat{\epsilon}_i) \chi_m(\vec{r}) \right]$$

where $\vec{q} = \vec{k}_i - \vec{k}_f$ is the momentum transfer. We recognize the integral as a Fourier transform,

$$\hat{\epsilon}_f^* \cdot \vec{A}_{sc}^{(i)} = D_0 \frac{k^2}{4\pi} \left[\underbrace{(\hat{\epsilon}_f^* \cdot \hat{\epsilon}_i) \chi_e(\vec{q})}_{\text{charge scattering}} + \underbrace{(\hat{k}_f \times \hat{\epsilon}_f^*) \cdot (\hat{k}_i \times \hat{\epsilon}_i) \chi_m(\vec{q})}_{\text{magnetic scattering}} \right]$$

Note that this expression gives the correct polarization dependence.

Often χ_e is a tensor. This happens in anisotropic

media. In optics the term for this is **linear birefringence**. In the x-ray business it is customary to bundle all these effects, including magnetic scattering, into a single, tensorial susceptibility, i.e.

$$\hat{\epsilon}_f^* \cdot \vec{A}_{sc}^{(i)} = D_0 \frac{k^2}{4\pi} \hat{\epsilon}_f^* \cdot \vec{\chi}(\vec{q}) \cdot \hat{\epsilon}_i$$

where $\vec{\chi}(\vec{q}) = (\chi_{ij})$ is a 3x3 matrix. This matrix may be decomposed into symmetric vs. antisymmetric components that are even or odd under time-reversal, respectively:

$$\chi_{ij} = \chi_{ij}^S + \chi_{ij}^A$$

$$\chi_{ij}^S = \frac{\chi_{ij} + \chi_{ji}}{2} \quad \text{even under time-reversal}$$

charge scattering

$$\chi_{ij}^A = \frac{\chi_{ij} - \chi_{ji}}{2} \quad \text{odd under time-reversal}$$

magnetic scattering

General comments:

Is the Born approximation good? Usually. In most circumstances (for x-rays) it's valid. However there are cases, eg. near strong resonances such as the La M_{4,5} edges, or in grazing-incidence reflectivity, where

Born breaks down. In such cases, **multiple scattering** must be taken into account. In such situations it is **crucial** to revert to classical scattering approaches, where multiple scattering effects may often be dealt with via familiar optical concepts such as **refraction** or **Fresnel reflectivity** via an approach called the **Distorted wave Born approx.**

How is classical scattering related to QM treatments using, e.g., the **Kramers-Heisenberg "formula"**? For elastic scattering they are **equivalent**. For suitable definition of $\chi(q)$, the two give identical answers. Which to use, then, depends on the circumstances. QM scattering is required, however, to relate χ to microscopic parameters.

Microscopic form for χ - classical power density

While the scattering can be understood classically, we still need to understand how χ is related to microscopics. This may be done by solving the QM scattering problem and comparing the two.

To compare QM and classical approaches we cannot just compare scattering amplitudes, which have completely different physical meaning. We must focus instead on a physical quantity. The best quantity to use is the energy flux (i.e. power, i.e. energy/time) flowing into the detector. We will compute this both classically and quantum mechanically and compare.

For simplicity we will assume χ here is a scalar and generalize the expressions when we are done.

Power in E&M is quantified by the Poynting vector, which for harmonic ($e^{-i\omega t}$) fields is

$$\vec{S} = \frac{1}{2} (\vec{E} \times \vec{H}^*) \quad (\text{time-averaged})$$

For a plane wave

$$\vec{S} = c u = \frac{c}{2\epsilon_0} |D_0|^2 \hat{k}$$

↑ ↑
speed of propagation energy density

If $\chi(q)$ is a scalar, the field at the detector is

$$\hat{\epsilon}_f^* \cdot \vec{D} = D_0 \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{4\pi r} (\hat{\epsilon}_f^* \cdot \hat{\epsilon}_i) k^2 \chi(q)$$

(Assuming you know how to block the direct beam).
This gives

$$S = \frac{c}{2\epsilon_0} D_0^2 \frac{1}{(4\pi r)^2} (\hat{\epsilon}_f^* \cdot \hat{\epsilon}_i)^2 k^4 |\chi(q)|^2$$

↑
Rayleigh's explanation of the blue sky.

This is the power density (energy/area/time). The integrated power flowing into the detector is

$$P = S \cdot A = S \cdot r^2 d\Omega$$

$$P = \frac{c}{2\epsilon_0} \left(\frac{P_0}{4\pi}\right)^2 (\epsilon_f \cdot \epsilon_i)^2 k^4 |\chi(q, \omega)|^2 d\Omega$$

This has units of watts. In gaussian units, this has the form

$$P = \frac{c}{2} D_0^2 (\epsilon_f \cdot \epsilon_i)^2 k^4 |\chi(q, \omega)|^2 d\Omega$$

Power Density for QM scattering

The entire concept of "elastic" scattering is more difficult to define quantum mechanically. Among other things, because the photon is quantized the scattering always depends on the photon density of final states, so one must discuss the power density scattered into a specified bandwidth. If the system is gapless, T is always non-negligible, and there is no sharp distinction between elastic and inelastic scattering. However we can make a direct comparison if we assume the system has an energy gap Δ . To ensure that all the measured scattering is elastic, we will assume the temperature $k_B T \ll \Delta$, and that the experiment employs an energy analyzer

so that only elastically scattered photons are detected.

For this part we will temporarily switch to gaussian units, which are more convenient.

In QM scattering, the focus of the discussion is the probability of transitioning from some initial state to some final state. In our case,

$$|i\rangle = a_{k_i, \lambda_i}^\dagger |0\rangle \quad |f\rangle = a_{k_f, \lambda_f}^\dagger |n\rangle \quad \lambda = \text{polarization index}$$

where a_k^\dagger creates a photon with wave vector k , and $|n\rangle$ is a many-body eigenstate of the electron system, $|0\rangle$ being the ground state. Here we have made use of the fact that $k_B T$ is small, in which case the Boltzmann factor

$$P_n = \frac{e^{-E_n/k_B T}}{Z} = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases}$$

Z
↑
partition function

In other words, the photon scatters from

initial momentum \vec{k}_i to final momentum \vec{k}_f , transitioning the system from its ground state $|0\rangle$ to some excited state, $|n\rangle$.

If electrons and photons did not interact with each other, the Hamiltonian of the system would be

$$\hat{H}^0 = \hat{H}_{EM}^0 + \hat{H}_e^0$$

where

$$\hat{H}_{EM}^0 = \int d^3r \left(\frac{\vec{E}^2}{2} + \frac{\vec{B}^2}{2} \right)$$

$$\hat{H}_e^0 = \int d^3r \hat{\psi}^\dagger(\vec{r}, t) \left[\frac{\vec{p}^2}{2m} + V(\vec{r}) \right] \hat{\psi}(\vec{r}, t)$$

The coupling between the two takes place via the Lorentz force law

$$m\ddot{\vec{r}} = -e \left(\vec{E} + \frac{1}{c} \dot{\vec{r}} \times \vec{B} \right)$$

In the Hamiltonian formalism, this law is expressed in terms of the Canonical momentum,

$$\vec{p}_c \equiv \frac{\partial \mathcal{L}}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} - \frac{e}{c} \vec{A}$$

where $\mathcal{L} = K - V = \frac{1}{2} m \dot{\vec{r}}^2 + e\phi - \frac{e}{c} \dot{\vec{r}} \cdot \vec{A}$ is the Lagrangian and \vec{A} is the second-quantized vector potential. Classically, \vec{A} is just a function. Here it is the field operator for the photons,

$$\hat{\vec{A}}(\vec{r}, t) = \sqrt{\frac{2\pi\hbar}{V}} \sum_{\vec{k}, \lambda} \frac{c}{\omega_{\vec{k}}} \left[a_{\vec{k}, \lambda} \hat{\vec{E}}_\lambda e^{i(\vec{k} \cdot \vec{r} - \omega t)} + h.c. \right]$$

Substituting into the expression for H_e gives

$$H_e = \int d^3r \hat{\psi}^\dagger(\vec{r}, t) \left[\frac{1}{2m} \left(\vec{p}_c + \frac{e}{c} \vec{A} \right)^2 + V \right] \hat{\psi}(\vec{r}, t)$$

Multiplying out the square gives

$$H_e = H_e^0 + H_1 + H_2$$

where

$$H_1 = \frac{e^2}{2mc^2} \int d^3r \hat{\psi}^\dagger(\vec{r}, t) \vec{A}(\vec{r}, t) \cdot \vec{p} \hat{\psi}(\vec{r}, t)$$

$$H_2 = \frac{e^2}{2mc^2} \int d^3r \vec{A}(\vec{r}, t)^2 \hat{\rho}(\vec{r}, t)$$

[we have assumed Coulomb gauge, $\nabla \cdot \vec{A} = 0$]

where $\hat{\rho}(\vec{r}, t) = \hat{\psi}^\dagger(\vec{r}, t) \hat{\psi}(\vec{r}, t)$ is an operator for the electron density.

Now that our electrons and photons interact, we can do scattering. This is

↑
annihilates
an electron
at \vec{r} at time
t.

done by computing the S matrix,

$$S_{f \leftarrow i} = \langle f | U_{\pm}(\infty, -\infty) | i \rangle$$

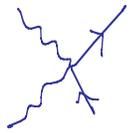
where the time evolution operator in the interaction picture

$$U_{\pm}(\infty, -\infty) = \exp \left[-\frac{i}{\hbar} \int_{-\infty}^{\infty} dt H_{\pm}(t) e^{-\gamma|t|} \right],$$

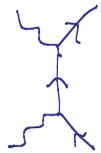
The γ serves to adiabatically switch our interaction on and off.

In principle this is an infinite **Dyson series**. Like the Rayleigh-Schrödinger case above, we will work in the first Born approximation. The lowest terms that contribute to scattering are

$$S_{f \leftarrow i} \approx \frac{-i}{\hbar} \int_{-\infty}^{\infty} dt \langle f | H_2(t) | i \rangle - \frac{1}{\hbar^2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \langle f | H_1(t_1) H_1(t_2) | i \rangle$$



This "seagull" diagram gives rise to instantaneous, **nonresonant scattering**.



This term proceeds via intermediate states and is the origin of **resonant scattering**.

Doing the time integrals,

$$S_{f \leftarrow i} = -\frac{2\pi i}{\hbar} \langle f | H_2(0) | i \rangle \delta(\omega_i - \omega_f - \omega_n)$$

$$+ \frac{2\pi}{\hbar} \sum_m \frac{\langle f | H_1(0) | m \rangle \langle m | H_1(0) | i \rangle}{E_i - E_m - i\gamma} \delta(\omega_f - \omega_i - \omega_n)$$

If we write out the matrix elements, we can see the physical meaning of each term. For the nonresonant case,

$$\langle f | H_2(0) | i \rangle = 2\pi \frac{\hbar e^2}{mV \sqrt{\omega_i \omega_f}} (\mathbf{E}_f^* \cdot \mathbf{E}_i) \langle n | \hat{\rho}(\mathbf{q}) | 0 \rangle$$

where $\vec{q} = \vec{k}_i - \vec{k}_f$ is the momentum transfer in the experiment. Hence, nonresonant scattering probes directly the electron density. If we consider only elastic processes (i.e., $|n\rangle = |0\rangle$) we call this **Thomson Scattering**, and precisely the same expression can be derived classically.

For the resonant case,

$$\langle f | H_1(0) | m \rangle \langle m | H_1(0) | i \rangle =$$

$$\frac{2\pi \hbar e^2}{mV} \frac{1}{\sqrt{\omega_i \omega_f}} \langle n | e^{-i\mathbf{k}_f \cdot \mathbf{r}} \mathbf{E}_f^* \cdot \vec{p} | m \rangle \langle m | e^{+i\mathbf{k}_i \cdot \mathbf{r}} \mathbf{E}_i \cdot \vec{p} | 0 \rangle$$

To get a transition rate we apply Fermi's Golden Rule:

$$\omega_{f \leftarrow i} = \frac{2\pi}{\hbar} \sum_n |M_{no}|^2 \delta(E_f - E_i + E_n)$$

where the total matrix element

$$M_{no} = -\langle f | H_2(o) | i \rangle + \frac{1}{i} \sum_m \frac{\langle f | H_1(o) | m \rangle \langle m | H_1(o) | i \rangle}{E_i - E_m - i\delta}$$

The differential scattering cross section

$$\frac{d^2\sigma}{d\Omega dE_f} = \frac{1}{\Phi} \omega_{f \leftarrow i} \frac{d^2N}{d\Omega dE_f}$$

where for photons in a box of volume V

$$\frac{d^2N}{d\Omega dE} = \frac{V \omega_f^2}{(2\pi)^3 \hbar c^3} \quad \Phi = \frac{c}{V}$$

This works out to be

$$= r_0^2 \frac{\omega_f}{\omega_i} (\epsilon_f^* \epsilon_i)^2 \times$$

$r_0 \equiv \frac{e^2}{mc^2}$ classical electron radius

$$\times \sum_n \left| -\langle n | \hat{p}(q) | 0 \rangle + \frac{1}{i(\epsilon_f \epsilon_i)} \sum_m \frac{\langle n | e^{-ik_f r} \epsilon_f^* \cdot \vec{p} | m \rangle \langle m | e^{+ik_i r} \epsilon_i | p \rangle}{E_i - E_m - i\delta} \right|^2$$

$$\times \delta(E_f - E_i + E_n)$$

We are interested in the power, but this depends on the range of scattered energy we measure. If we have an energy-resolving detector, we can isolate the elastic scattering.

$$P = \hbar\omega \cdot \Phi \cdot d\Omega \int_{E_f - \Delta}^{E_f + \Delta} \frac{d^2\sigma}{d\Omega dE_f} dE_f$$

where $2\Delta \ll \Delta$ is the energy bandwidth measured by our detector. Now, only purely elastic processes contribute:

$$P = \hbar\omega \frac{c}{V} d\Omega r_0^2 (\epsilon_f^* \epsilon_i)^2$$

$$\times \left| -\rho(q) + \frac{1}{i(\epsilon_f \epsilon_i)} \sum_m \frac{\langle 0 | e^{-ik_f r} \epsilon_f^* \cdot \vec{p} | m \rangle \langle m | e^{+ik_i r} \epsilon_i | p \rangle}{E_i - E_m - i\delta} \right|^2$$

↑
ground state electron density

We learn a great deal if we compare this to the classical result:

$$P = \frac{c}{2} D_0^2 (\epsilon_f \epsilon_i)^2 k^4 |\chi(q, \omega)|^2 d\Omega$$

Unlike the classical expression, which buries all the physics of the system inside a

susceptibility, $\chi(q, \omega)$, the QM expression is written in terms of microscopic operators for the electron system. Recognizing that

$$\frac{c}{2} |D^0|^2 = \pi \omega \Phi$$

where

$$\Phi = \frac{c}{v}$$

equating the two expressions gives a form for the polarizability:

$$\chi(q) = -\frac{r_0 \lambda^2}{4\pi^2} p(q) \quad \leftarrow \text{Thompson scattering}$$

$$+ \frac{r_0 \lambda^2}{4\pi^2 \epsilon_f \epsilon_i} \sum_m \frac{\langle 0 | e^{-ik_f \cdot r} \epsilon_f^* \cdot \vec{p} | m \rangle \langle m | e^{+ik_i \cdot r} \epsilon_i \cdot \vec{p} | 0 \rangle}{E_i - E_m - i\gamma} \quad \leftarrow \text{resonant scattering}$$

To get SI units, just multiply by $4\pi\epsilon_0$:

$$\chi(q) = -\frac{r_0 \lambda^2}{\pi} p(q)$$

$$+ \frac{r_0 \lambda^2}{\pi \epsilon_f \epsilon_i} \sum_m \frac{\langle 0 | e^{-ik_f \cdot r} \epsilon_f^* \cdot \vec{p} | m \rangle \langle m | e^{+ik_i \cdot r} \epsilon_i \cdot \vec{p} | 0 \rangle}{E_i - E_m - i\gamma}$$

The beauty of this expression is that it separates the optics part of the problem from the microscopic, ktt physics. Microscopic models can then focus on computing χ and experimenters can focus on measuring this.

From this expression we can also make several important observations:

1. Thompson scattering contributes to the polarizability of the system. Even if the resonant term is small (e.g., when the photon energy is far from any atomic transition) there is still a significant contribution to χ that cannot always be ignored. This contribution is always negative, so generically $n = \sqrt{\epsilon} < 1$ for x-rays.

2. The Thompson term is energy-dependent. Even without resonance processes, there is an intrinsic λ^2 dependence from Rayleigh's Law. This is the origin of what is called the normal dispersion of optical constants.

3. Despite #2, there is no intrinsic energy dependence to Thomson scattering. The λ^2 term in χ cancels that in Rayleigh's law. [The question of why the sky is blue, then,

is more subtle than it may seem.]

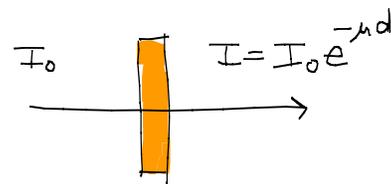
4. Resonant processes cause departures from simple λ^2 dependence. In the old days this was called **anomalous dispersion**. For this reason, resonant elastic scattering is sometimes called **anomalous scattering**.

5. The expression for resonant scattering is called the **Kramers-Heisenberg formula**. This is an unfortunate name. There are no "formulas" in science - or in life - that we may blindly apply without thought or introspection. The KH formula neglects multiple scattering, Thompson processes, etc. In cases where it applies, however, it is a quite general description of scattering in terms of many-body intermediate states of the valence electron system.

6. All the important physics about valence band phenomena is contained in the KH term. Because it in principle involves transitions into the valence band, the resonant term contains info about, e.g., spin structure, charge and orbital order, etc.

7. The local approximation in scattering is guaranteed to be valid for Thomson scattering, but not the KH term. It only works when the MFP of the excitations is less than $1/q$. Locality probably does not apply, for example, in the manganites.

X-ray absorption



$$\mu = \frac{1}{d} \log \frac{I_0}{I} = 2kk' = \frac{4\pi}{\lambda} k$$

$$k = \text{Im } n = \text{Im } \sqrt{\epsilon}$$
$$= \text{Im } \sqrt{1 + \chi} \approx \frac{1}{2} \text{Im } \chi$$

$$\text{Im } \chi \propto \text{Im} \sum_n \frac{|\langle m | e \cdot p e^{+ik \cdot r} | 0 \rangle|^2}{\hbar \omega_i - E_m - i\delta}$$

$$\propto \sum_m |\langle m | \hat{\epsilon} \cdot p e^{ik \cdot r} | 0 \rangle|^2 \delta(\hbar \omega_i - E_m)$$

usual formula for computing XAS

① Note: The Thomson cross section does not affect the XAS spectrum. So there is a tendency for spectroscopists to ignore it. But it can be strong, e.g. on a Bragg peak, even in highly resonant soft x-ray scattering.

② Note: XAS and anomalous scattering are intimately related. Can extract χ from XAS using Kramers-Kronig analysis. But don't forget to put in the Thomson part!

Atomic form factor

If we have a medium made of point scatterers, the density

$$\rho(\vec{r}) = \sum_n f_n \delta(\vec{r} - \vec{r}_n)$$

$f_n = Z$ = number of electrons on atom n .

What is the susceptibility?

$$\chi(\vec{q}) = -\frac{re\lambda^2}{\pi} \int d^3r \rho(\vec{r}) e^{-i\vec{q}\cdot\vec{r}} = -\frac{re\lambda^2}{\pi} \sum_n f_n e^{-i\vec{q}\cdot\vec{r}_n}$$

f is called an atomic scattering factor. While it was defined in terms of Thomson scattering, the concept is used in the resonant case also, where Kramers-Kronig effects are captured by the anomalous terms, i.e.,

$$f = Z + \underbrace{\Delta f'(\omega) + i\Delta f''(\omega)}_{\text{resonant term}}$$

↑
Thomson

This framework has the very important implication that it is possible to use XAS to measure the anomalous scattering factors needed to interpret

scattering experiments. That is,

$$\begin{aligned} \mu(\omega) &= \frac{4\pi}{\lambda} k(\omega) = \frac{2\pi}{\lambda} \text{Im} \chi(\omega) \Big|_{\vec{q}=0} \\ &= -2re\lambda \sum_n \text{Im} f_n(\omega) \\ &= -2re\lambda \sum_n \Delta f_n''(\omega) \end{aligned}$$

From $\Delta f_n''(\omega)$, one can—in principle—calculate $\Delta f_n'(\omega)$ from the Kramers-Kronig relations to achieve a complete parameterization of the full, complex $f(\omega)$.

Periodic Systems: the Structure Factor

We will finish our discussion by addressing the question of scattering from a periodic system, which is almost always what we are doing. This will demonstrate an important connection between absorption and scattering experiments.

$$\rho(\vec{r}) = \sum_n f_n \delta^3(\vec{r} - \vec{r}_n) = \sum_R \sum_n f_n \delta^3(\vec{r} - \vec{R} - \vec{r}_n)$$

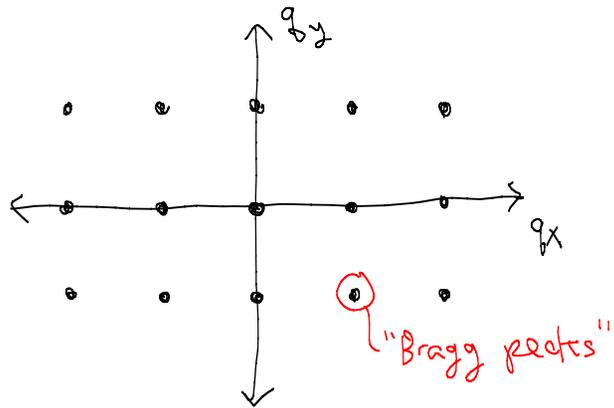
↑
all atoms in the solid

$$\rho(\vec{r}) = \sum_{\vec{G}} \rho_G e^{i\vec{G} \cdot \vec{r}} \quad \text{Fourier series}$$

$$\rho_G = \frac{1}{V_{\text{cell}}} \int_{\text{cell}} d\vec{r} \rho(\vec{r}) e^{-i\vec{G} \cdot \vec{r}}$$

$$= \frac{1}{V_{\text{cell}}} \sum_n f_n e^{-i\vec{G} \cdot \vec{r}_n} = \frac{1}{V_{\text{cell}}} S(\vec{q})$$

↑
structure factor



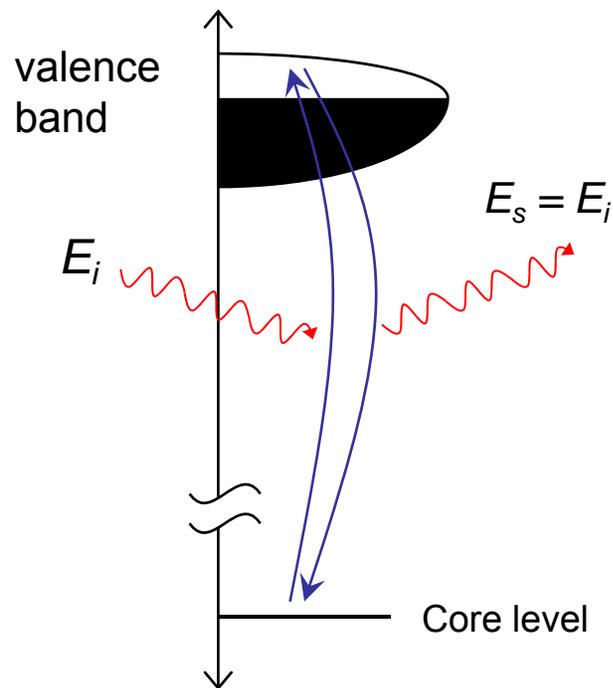
Scattering localized in spots, each

corresponding to a unique \vec{G} . The integrated intensity of each is proportional to $V|\rho_G|^2$.

Suggests the following prescription:

1. Use XAS to calibrate f 's
2. Use to interpret scattering data.

Resonant (elastic) scattering: main points



- Resonant x-ray scattering is a probe of the **ground state** valence electronic structure
- Optimized for studying phenomena that are **electronically inhomogeneous**:
- Antiferromagnetism, spin density waves, charge stripes, Wigner crystals, orbital order, toroidal order, electronic reconstruction of interfaces, etc.
- Closest sibling is STM
- **Easier to interpret than RIXS**. Has made a **bigger impact** in condensed matter physics.
- **Still some subtleties** to understand – this is a good thing

Origins: Resonant magnetic hard x-ray scattering



VOLUME 61, NUMBER 10

PHYSICAL REVIEW LETTERS

5 SEPTEMBER 1988

Polarization and Resonance Properties of Magnetic X-Ray Scattering in Holmium

Doon Gibbs

Brookhaven National Laboratory, Upton, New York 11973

D. R. Harshman, E. D. Isaacs, and D. B. McWhan

AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974

D. Mills

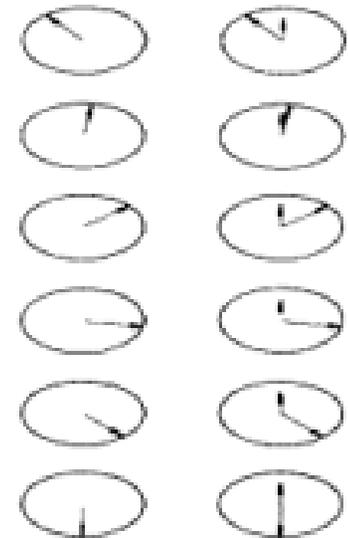
Cornell High Energy Synchrotron Source, Cornell University, Ithaca, New York 148.

and

C. Vettier

Institut Laue-Langevin, 38042 Grenoble, Cedex, France

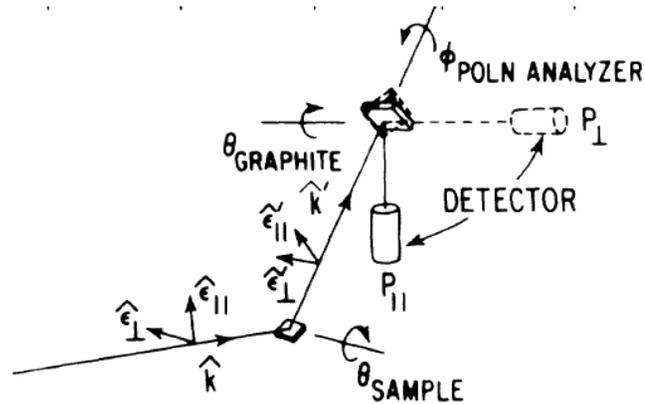
(Received 15 June 1988)



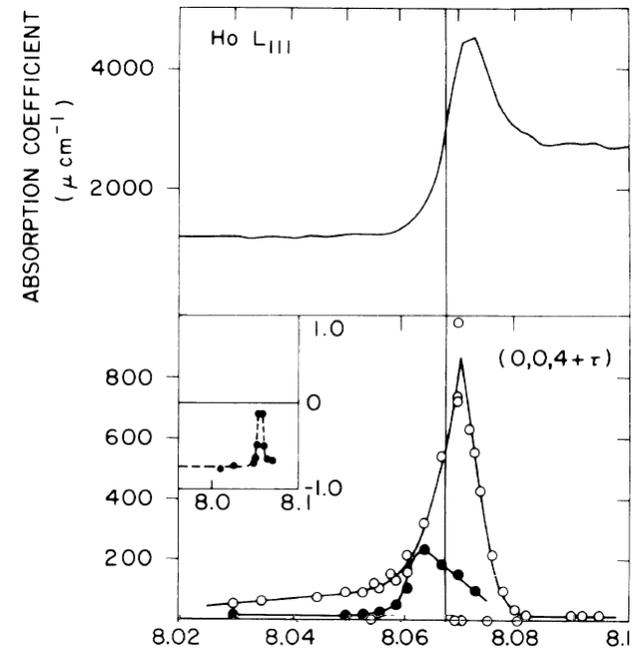
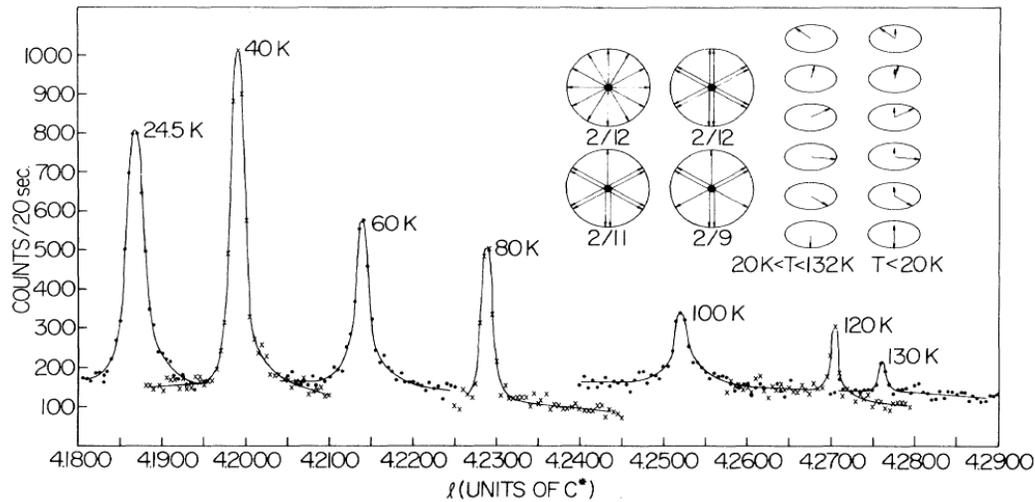
20K < T < 132K T < 20K

- Holmium metal
- HCP crystal structure
- Spiral spin density wave (SDW) of FM aligned layers along c axis
- Ho L_3 edge at 7.85 keV was accessible at CHESS and SSRL
- $2p \rightarrow 5d$ transition goes directly to the $4f / 5d$ valence band
- Lots of spin-orbit coupling \Rightarrow large magnetic scattering cross section

Origins: resonant magnetic scattering



$$\epsilon_f \cdot A_{sc}^{(c)} = D_0 \frac{k^2}{4\pi^2} \left[\underbrace{(\hat{\epsilon}_f^* \cdot \hat{\epsilon}_i) \chi_e(\vec{q})}_{\text{charge scattering}} + \underbrace{(\hat{k} \times \hat{\epsilon}_f^*) \cdot (\hat{k}_i \times \hat{\epsilon}_i) \chi_M(\vec{q})}_{\text{magnetic scattering}} \right]$$



- Peak only visible on resonance
- Exhibits expected polarization dependence

Modern Era: Soft X-Rays

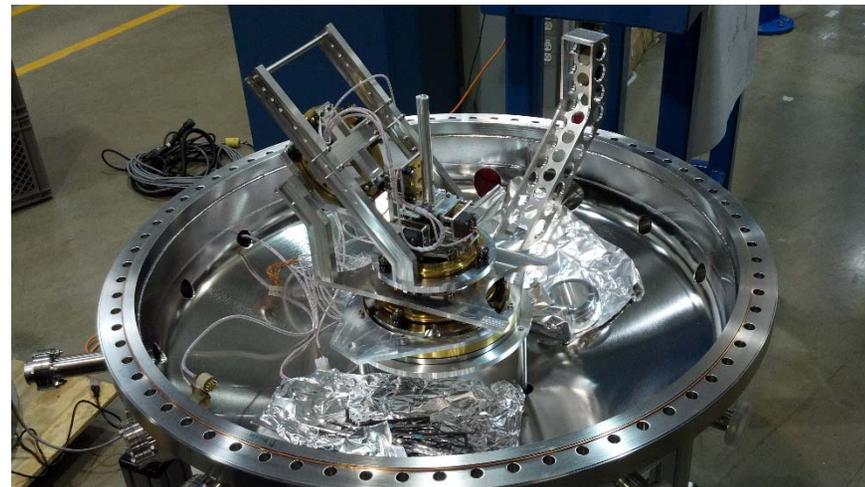
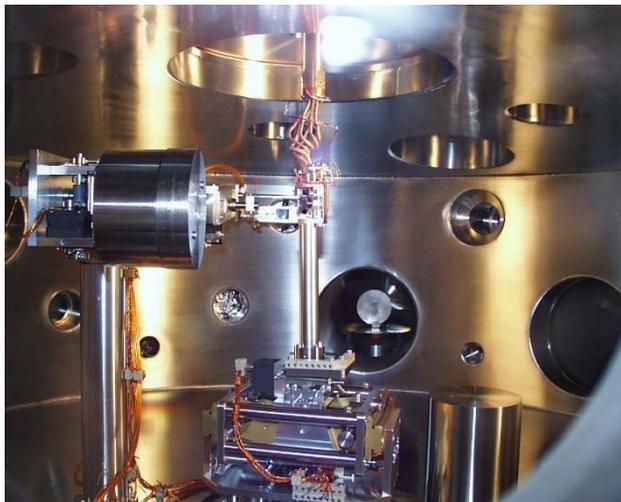
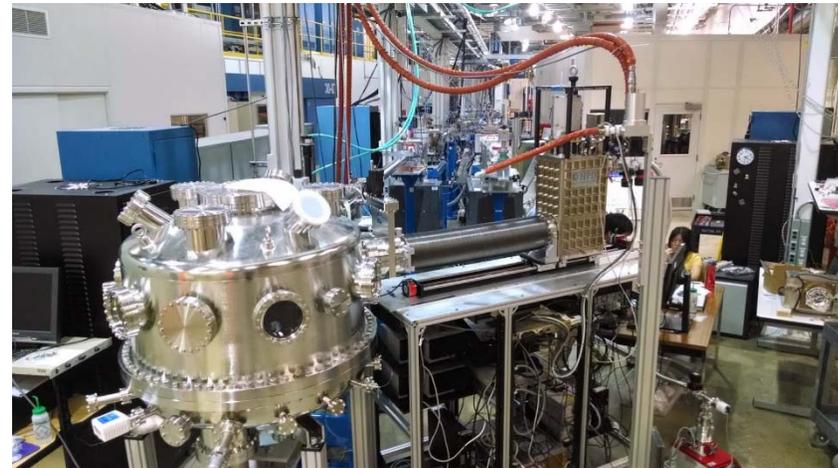
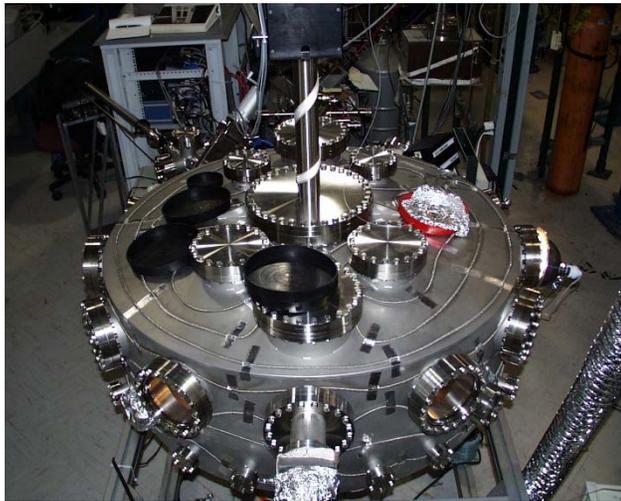


- The Gibbs experiment was possible because the Ho L_3 edge is in the hard x-ray regime: Accessible with existing facilities at CHESS, SSRL
- Problem: **Rare earth materials generally do not order.** Mainly of interest in CMP for Kondo lattice and Heavy Fermion physics, which are spatially homogeneous phenomena (great for DMFT).
- Inhomogeneity most prevalent in 3d transition metals: HTSC in cuprates and Fe-based materials, manganites, nickelates, etc.
- These materials have TM 3d – O 2p hybridized valence bands. Relevant edges are in the soft x-ray range:
 - ◇ Oxygen K edge (1s-2p): 540 eV
 - ◇ 3d TM L edges: 480 – 950 eV (Ti through Cu)
- Requires scattering with soft x-rays. Everything has to be in UHV.

Modern Era: Soft X-Rays



- Field was enabled by development of vacuum diffractometers



NSLS X1B

Sector 29 at APS (new in 2014)

Soft x-ray scattering from iron foil



VOLUME 65, NUMBER 3

PHYSICAL REVIEW LETTERS

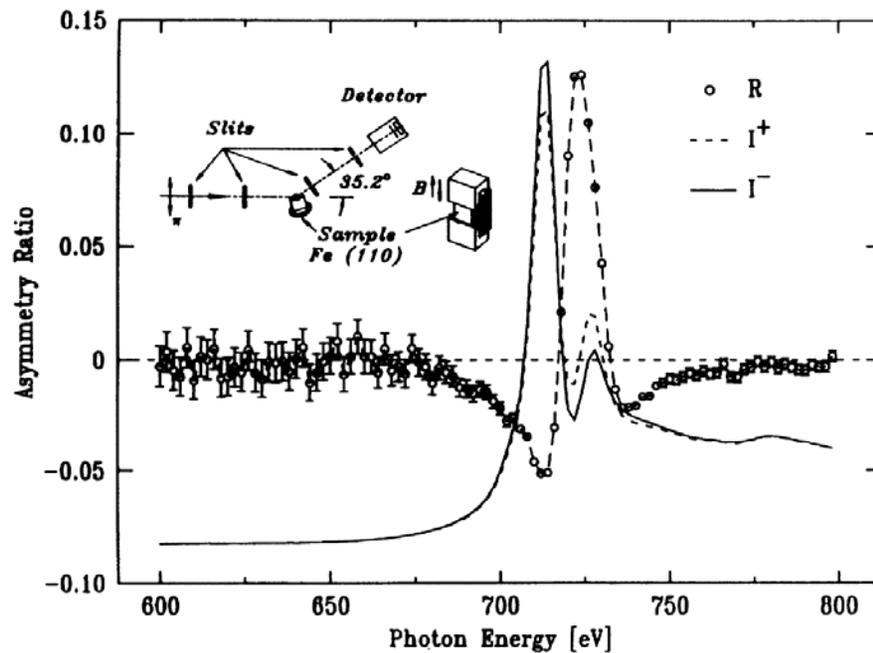
16 JULY 1990

Magnetic-Resonance Exchange Scattering at the Iron L_{II} and L_{III} Edges

C. Kao, J. B. Hastings, E. D. Johnson, D. P. Siddons, and G. C. Smith
 Brookhaven National Laboratory, Upton, New York 11973

G. A. Prinz

Naval Research Laboratory, Washington, D.C. 20375
 (Received 25 January 1990)



- Specular reflectance from an iron film.
- Ferromagnetism contributes to scattering at $q=0$
- Line shape at Fe L3 edge indicates interference between Thomson and resonant scattering

$$P \sim \left| -p(q) + \frac{1}{i(f_f \cdot \epsilon_i)} \sum_m \frac{\langle 0 | e^{-i\mathbf{k}_f \cdot \mathbf{r}} \epsilon_f^* \cdot \hat{p} | m \rangle \langle m | e^{+i\mathbf{k}_i \cdot \mathbf{r}} \epsilon_i \cdot \hat{p} | 0 \rangle}{E_i - E_m - i\gamma} \right|^2$$

↑
ground state
electron density

Measure the orbital state in manganites?



PHYSICAL REVIEW B

VOLUME 62, NUMBER 2

1 JULY 2000-II

Orbital ordering in the manganites: Resonant x-ray scattering predictions at the manganese L_{II} and L_{III} edges

C. W. M. Castleton*

European Synchrotron Radiation Facility, BP 220, F-38043 Grenoble Cédex, France

M. Altarelli

Sincrotrone Trieste, Area Science Park, 34012 Basovizza, Trieste, Italy

and Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy

(Received 29 November 1999; revised manuscript received 16 February 2000)

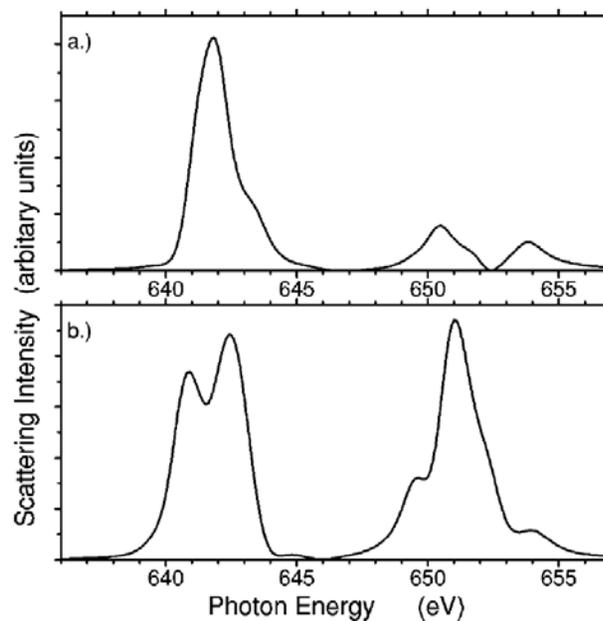
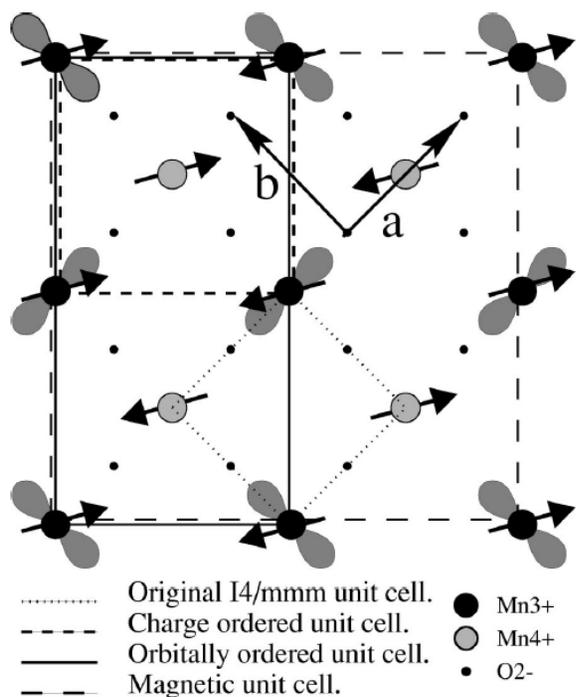


FIG. 4. (a) Calculated intensity at the Mn $L_{II(III)}$ edges. (b) Calculated intensity with signs of X^{420} and X^{220} reversed.

- Prediction for $LaMnO_3$
- Used atomic multiplet approach for computing $f(\omega)$



A Structural Probe of the Doped Holes in Cuprate Superconductors

P. Abbamonte,^{1,2,*†} L. Venema,¹ A. Rusydi,¹ G. A. Sawatzky,^{1‡}
G. Logvenov,³ I. Bozovic³

www.sciencemag.org SCIENCE VOL 297 26 JULY 2002

- Use XAS and Kramers-Kronig to compute form factors for $\text{La}_2\text{CuO}_{4+y}$
- Scattering from doped holes enhanced by more than 10^4 at O K edge.
- Direct detection of valence band order in cuprates possible: Stripes, checkerboards, etc.

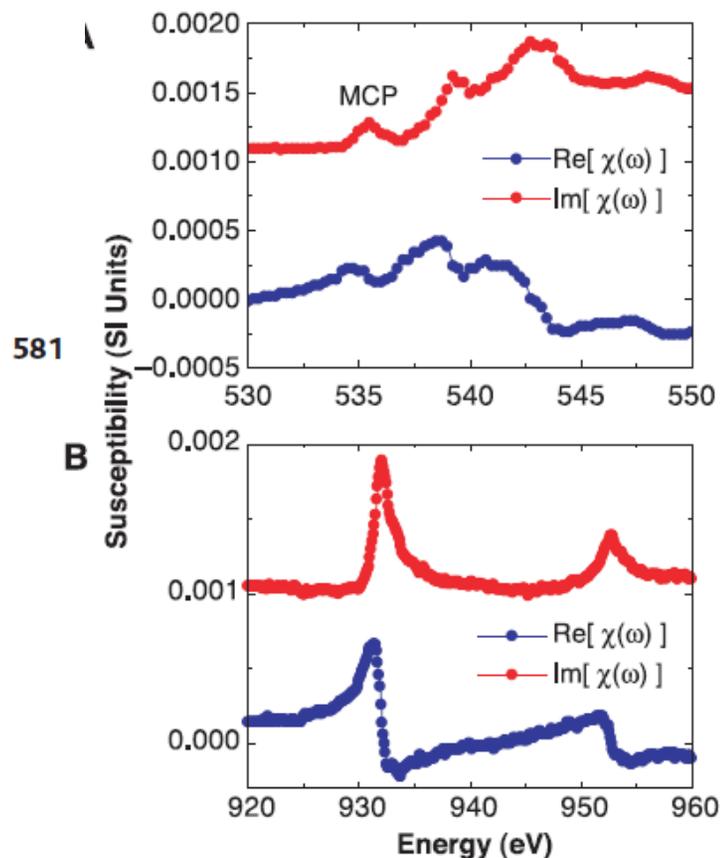


Fig. 2. ab -plane susceptibility of optimally doped $\text{La}_2\text{CuO}_{4+\delta}$, in SI units, calculated by the procedure described in the vicinity (A) of the OK edge and (B) of the $\text{CuL}_{2,3}$ edge.

Magnetic scattering: bilayer manganite



VOLUME 90, NUMBER 18

PHYSICAL REVIEW LETTERS

week ending
9 MAY 2003

Soft X-Ray Resonant Magnetic Diffraction

S. B. Wilkins* and P. D. Hatton†

Department of Physics, University of Durham, Rochester Building, South Road, Durham DH1 3LE, United Kingdom

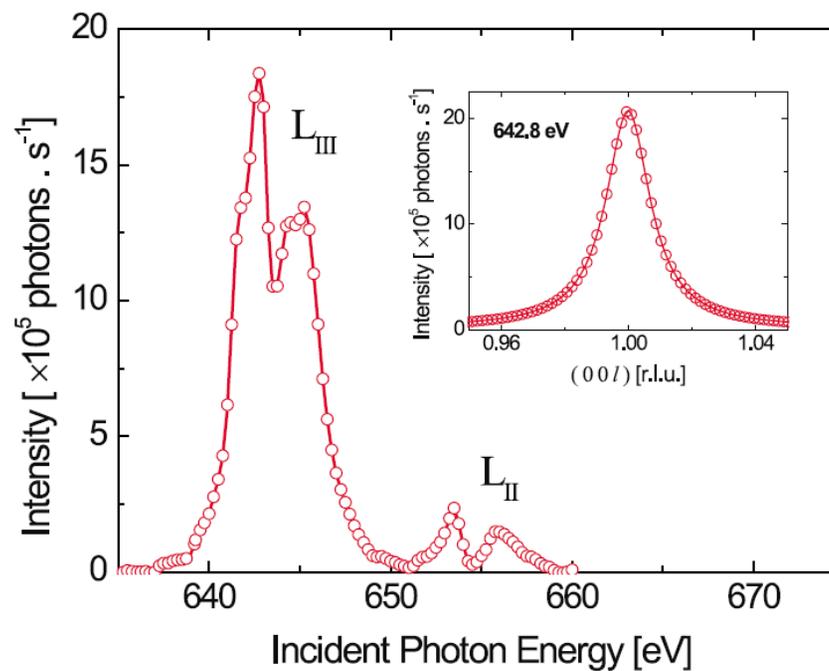
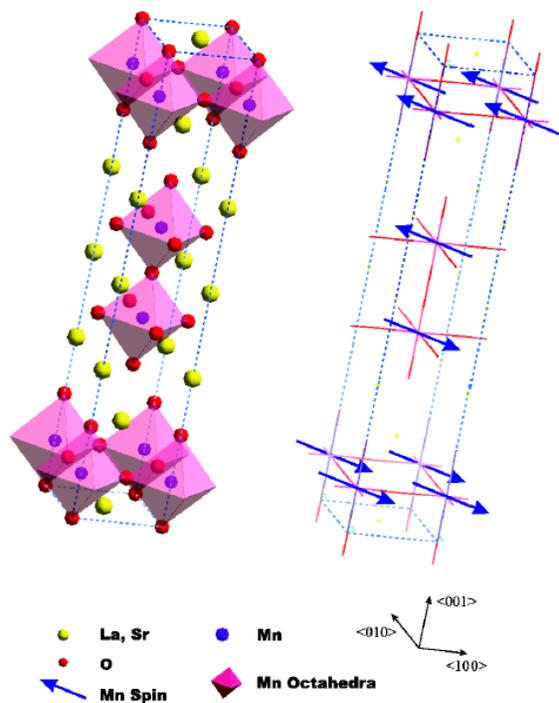
M. D. Roper

CLRC, Daresbury Laboratory, Warrington, Cheshire WA4 4AD, United Kingdom

D. Prabhakaran and A. T. Boothroyd

Department of Physics, University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, United Kingdom

(Received 28 January 2003; published 8 May 2003)



Orbital Scattering



VOLUME 91, NUMBER 16

PHYSICAL REVIEW LETTERS

week ending
17 OCTOBER 2003

Direct Observation of Orbital Ordering in $\text{La}_{0.5}\text{Sr}_{1.5}\text{MnO}_4$ Using Soft X-ray Diffraction

S. B. Wilkins,* P. D. Spencer, and P. D. Hatton

Department of Physics, University of Durham, Rochester Building, South Road, Durham, DH1 3LE, United Kingdom

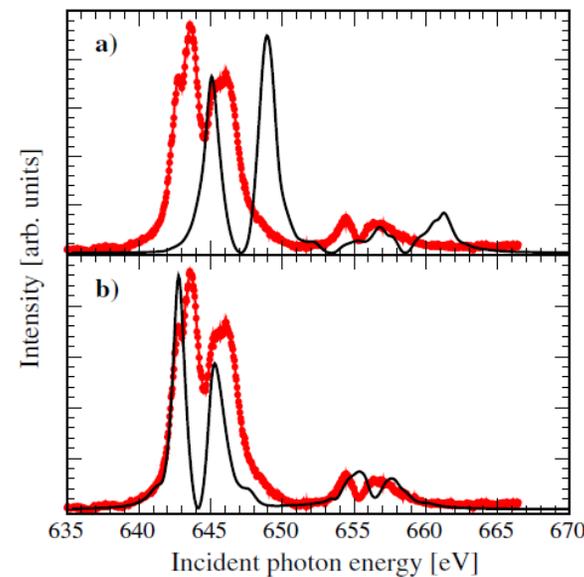
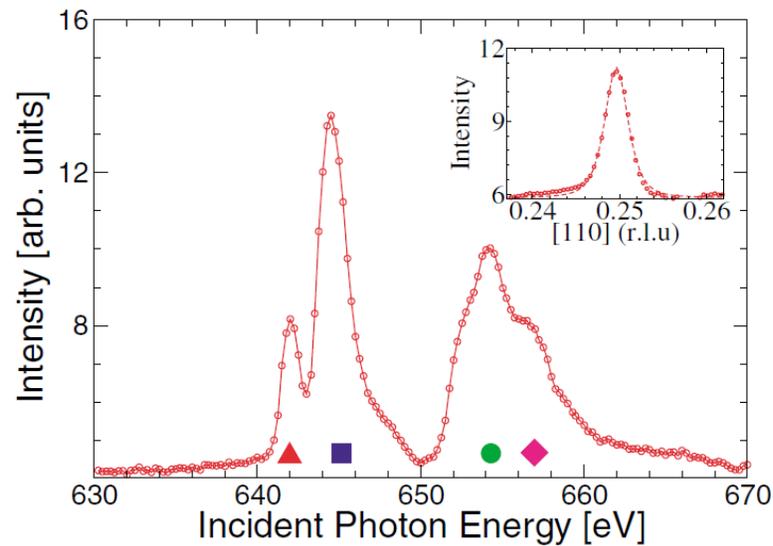
S. P. Collins[†] and M. D. Roper

Daresbury Laboratory, Warrington, Cheshire WA4 4AD, United Kingdom

D. Prabhakaran and A. T. Boothroyd

Department of Physics, University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, United Kingdom

(Received 24 April 2003; published 16 October 2003)



- Could never get good enough agreement with theory to deduce orbital shape.
- Too much nonlocality?

Focus on Polarization / Symmetry



PHYSICAL REVIEW B 71, 214421 (2005)

Orbital and magnetic ordering in $\text{La}_{0.5}\text{Sr}_{1.5}\text{MnO}_4$ studied by soft x-ray resonant scattering

U. Staub,¹ V. Scagnoli,¹ A. M. Mulders,¹ K. Katsumata,² Z. Honda,³ H. Grimmer,⁴ M. Horisberger,⁴ and J. M. Tonnerre⁵

¹Swiss Light Source, Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland

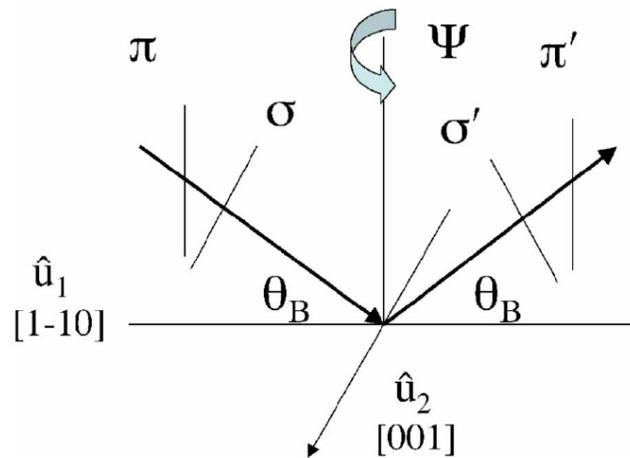
²RIKEN Harima Institute, Mikazuki, Sayo, Hyogo 679-5148, Japan

³Department of Functional Materials Science, Saitama University, Shimo-Ohkubo 255, Saitama, 338-8570, Japan

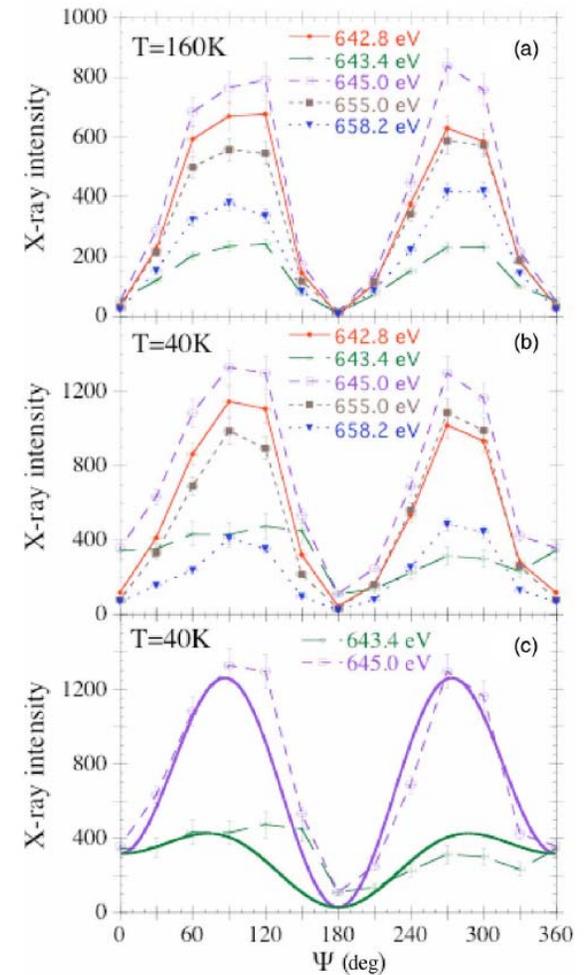
⁴Laboratory for Neutron Scattering, ETH & PSI, CH-5232 Villigen PSI, Switzerland

⁵Laboratoire de Cristallographie, CNRS, F-38042 Grenoble, France

(Received 25 March 2005; published 24 June 2005)



The $(1/4, 1/4, 0)$ reflection shows a single tensorial contribution between $T_{OO} > T > T_N$ reflecting the quadrupole of the $3d$ shell and an additional contribution below T_N . The latter is of magnetic origin and is not an indication of an enhanced Jahn-Teller distortion.



Wigner crystal in Spin Ladders



Crystallization of charge holes in the spin ladder of $\text{Sr}_{14}\text{Cu}_{24}\text{O}_{41}$

P. Abbamonte^{1,2}, G. Blumberg³, A. Rusydi^{1,4}, A. Gozar^{3,5}, P. G. Evans⁶, T. Siegrist³, L. Venema⁴, H. Eisaki⁷, E. D. Isaacs^{3,8} & G. A. Sawatzky⁹

¹National Synchrotron Light Source, Brookhaven National Laboratory, Upton, New York 11973, USA

²Department of Physics and Astronomy, SUNY Stony Brook, Stony Brook, New York 11794, USA

³Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA

⁴University of Groningen, 9747 AG Groningen, The Netherlands

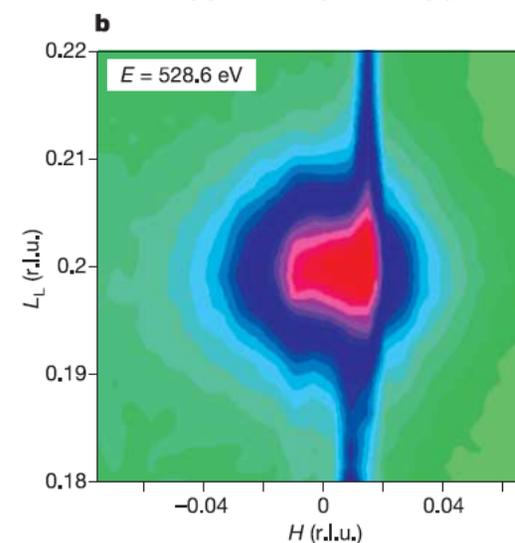
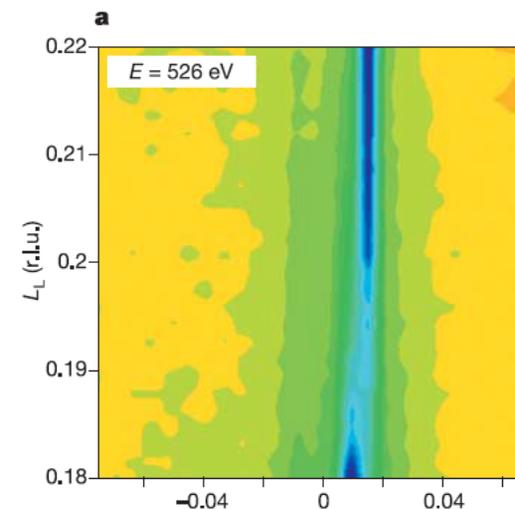
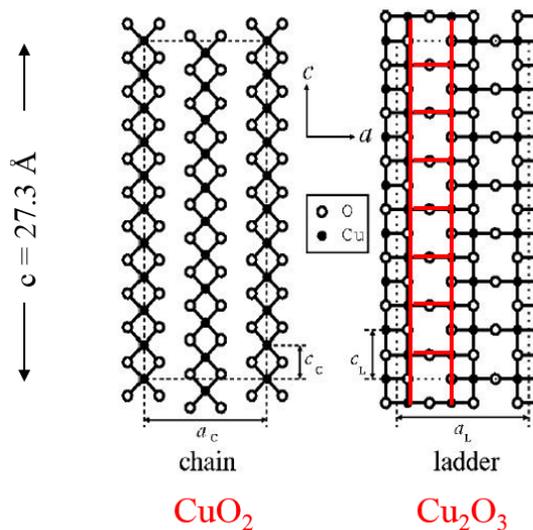
⁵Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

⁶Department of Materials Science & Engineering, University of Wisconsin, Madison, Wisconsin 53706, USA

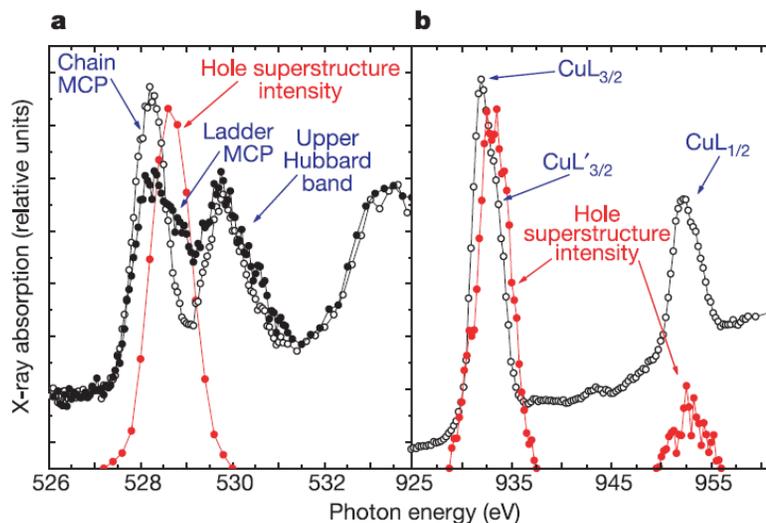
⁷Nanoelectronics Research Institute, AIST, 1-1-1 Central 2, Umezono, Tsukuba, Ibaraki, 305-8568, Japan

⁸Center for Nanoscale Materials, Argonne National Laboratory, Argonne, Illinois 60439, USA

⁹Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T-1Z1, Canada



NATURE | VOL 431 | 28 OCTOBER 2004 | www.nature.com/nature



Stripes in cuprates



LETTERS

Spatially modulated 'Mottness' in $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$

P. ABBAMONTE^{1,2*}, A. RUSYDI^{1,3†}, S. SMADICI¹, G. D. GU¹, G. A. SAWATZKY^{3,4} AND D. L. FENG⁵

¹National Synchrotron Light Source and Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA

²Department of Physics, University of Illinois, Urbana, Illinois 61801, USA

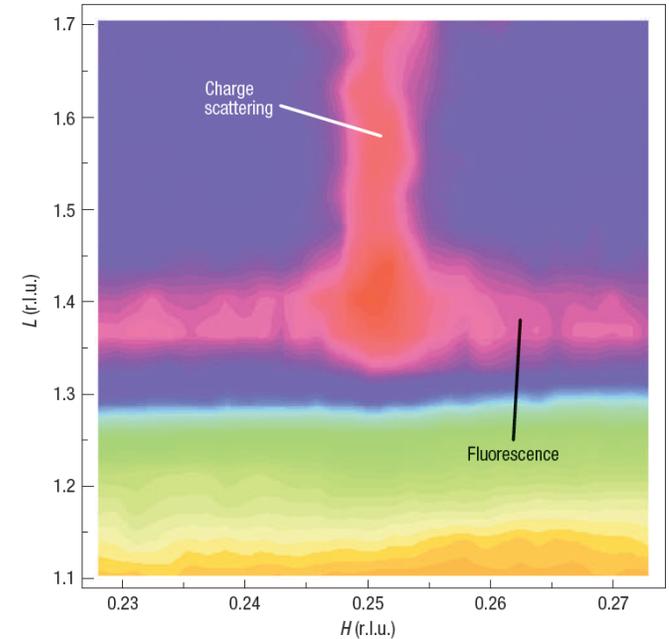
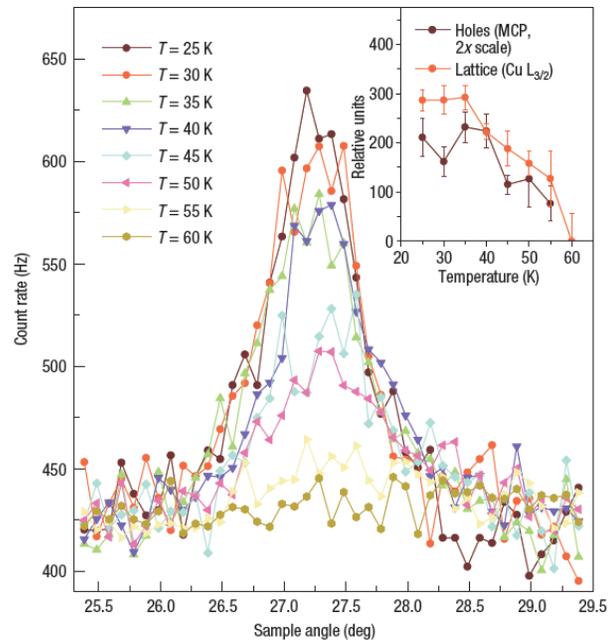
³Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands

⁴Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T-1Z1, Canada

⁵Department of Physics, Fudan University, Shanghai 200433, China

[†]Current address: Institute for Applied Physics, University of Hamburg, D-20355 Hamburg, Germany

*e-mail: abbamonte@mri.uiuc.edu



- Calibrated $f(\omega)$ with x-ray absorption
- Deduced charge amplitude ~ 0.5 holes: "stripes are charged"
- Close to expectation for half-filled stripes
- Looked at YBCO and BSCCO. Never saw anything. Gave up.



Electronic Reconstruction at SrMnO₃-LaMnO₃ Superlattice Interfaces

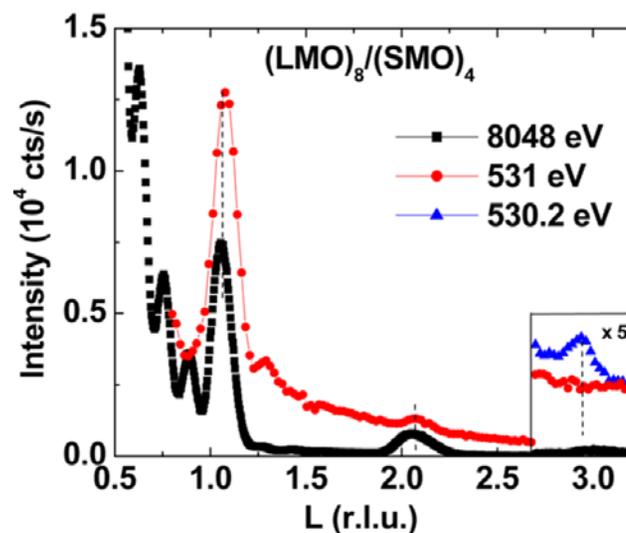
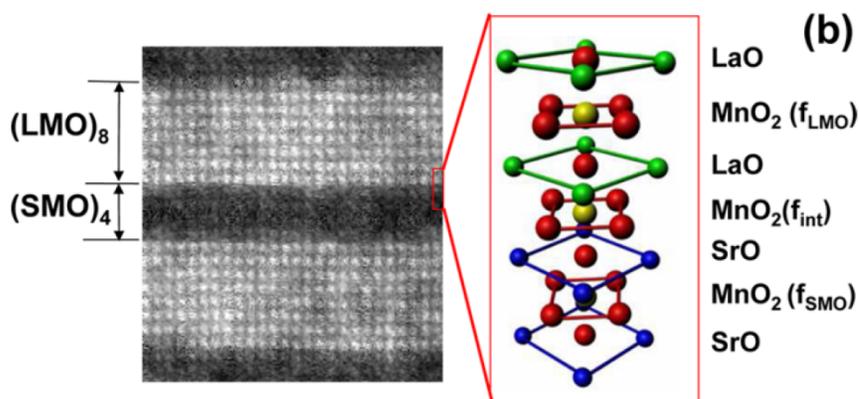
Şerban Smadici,¹ Peter Abbamonte,¹ Anand Bhattacharya,² Xiaofang Zhai,¹ Bin Jiang,¹ Andriwo Rusydi,³
James N. Eckstein,¹ Samuel D. Bader,² and Jian-Min Zuo¹

¹*Frederick Seitz Materials Research Laboratory, University of Illinois, Urbana, Illinois 61801, USA*

²*Center for Nanoscale Materials and Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

³*Institute of Applied Physics, University of Hamburg, D-20355 Hamburg, Germany*

(Received 31 May 2007; published 9 November 2007)



$$S(L = 3) = 2f_{int} - f_{LMO} - f_{SMO}$$

- Deduced existence of Fermi surface at Mott-Band insulator interface
- In retrospect, it may have been magnetic scattering

Heterostructures



NATURE MATERIALS | LETTER



Orbital reflectometry of oxide heterostructures

Eva Benckiser, Maurits W. Haverkort, Sebastian Brück, Eberhard Goering, Sebastian Macke, Alex Frañó, Xiaoping Yang, Ole K. Andersen, Georg Cristiani, Hanns-Ulrich Habermeier, Alexander V. Boris, Ioannis Zegkinoglou, Peter Wochner, Heon-Jung Kim, Vladimir Hinkov & Bernhard Keimer

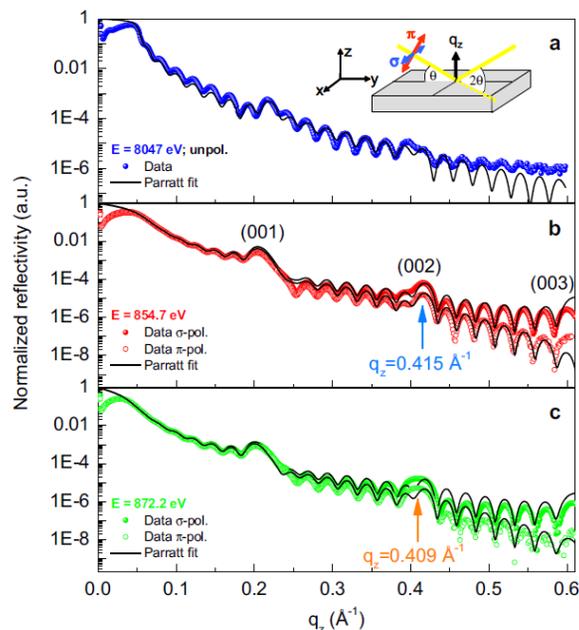
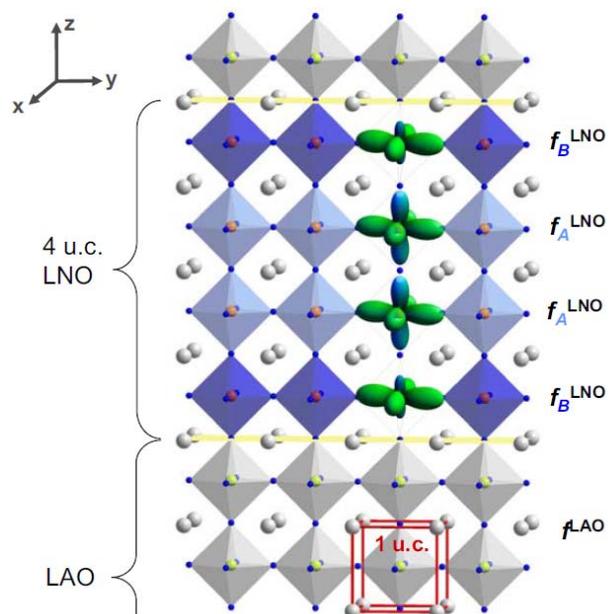
Affiliations | Contributions | Corresponding authors

Nature Materials 10, 189–193 (2011) | doi:10.1038/nmat2958

Received 25 August 2010 | Accepted 07 January 2011 | Published online 06 February 2011

$$\frac{n_{3z^2-r^2}}{n_{x^2-y^2}} = \frac{3I_{E||z}}{4I_{E||x} - I_{E||z}}$$

$$P = \frac{(n_{x^2-y^2} - n_{3z^2-r^2})}{(n_{x^2-y^2} + n_{3z^2-r^2})}$$



- Measured orbital disproportionation between inner and outer NiO₂ layers
- “Orbital engineering” with oxide MBE

Loop currents (“anapoles”) in CuO



Scienceexpress

Report

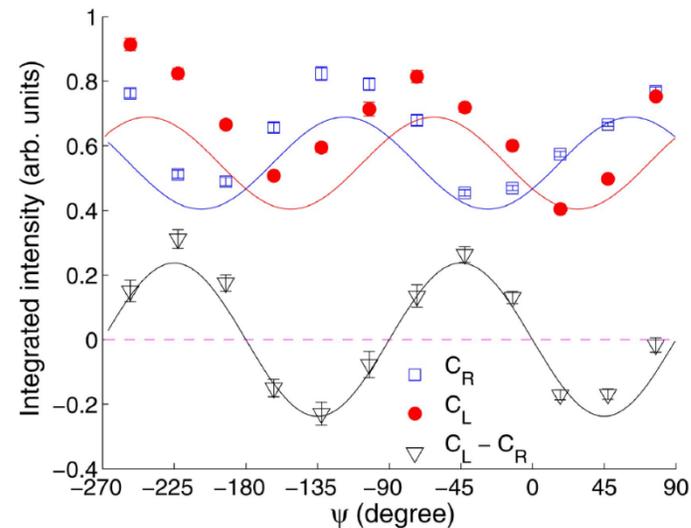
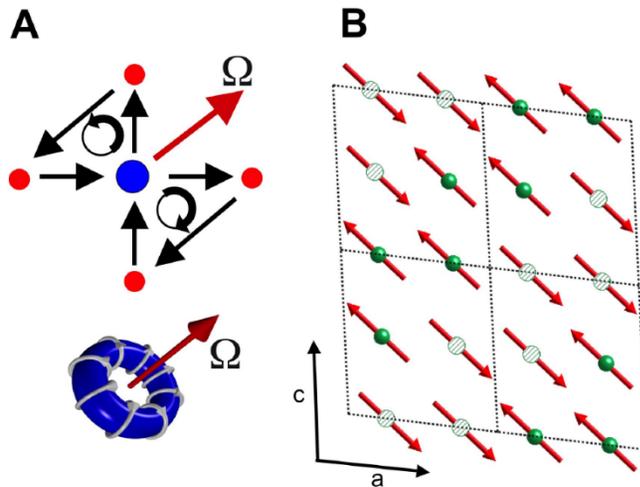
Observation of Orbital Currents in CuO

V. Scagnoli,^{1*} U. Staub,¹ Y. Bodenthin,¹ R. A. de Souza,¹ M. García-Fernández,¹ M. Garganourakis,¹ A. T. Boothroyd,² D. Prabhakaran,² S. W. Lovesey^{3,4}

¹Swiss Light Source, Paul Scherrer Institut, CH 5232 Villigen PSI, Switzerland. ²Department of Physics, University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK. ³ISIS Facility, RAL, Oxfordshire OX11 0QX, UK ⁴Diamond Light Source Ltd., Oxfordshire OX11 0QX, UK

*To whom correspondence should be addressed. E-mail: valerio.scagnoli@psi.ch

$$\text{Toroidal moment } \langle \Omega \rangle = \langle \mathbf{R} \times \mathbf{S} \rangle \neq \langle \mathbf{R} \rangle \times \langle \mathbf{S} \rangle$$



Charge order in YBCO

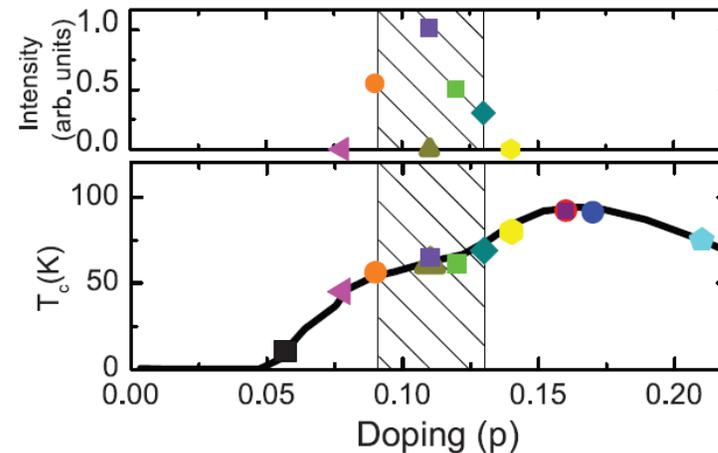
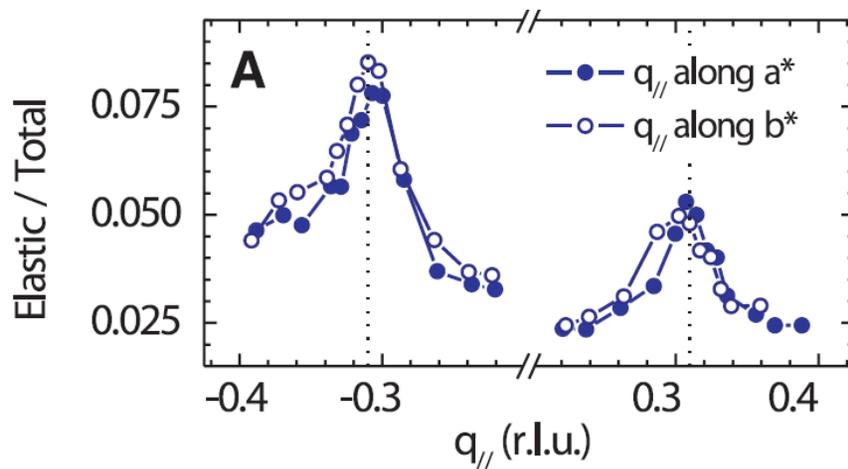
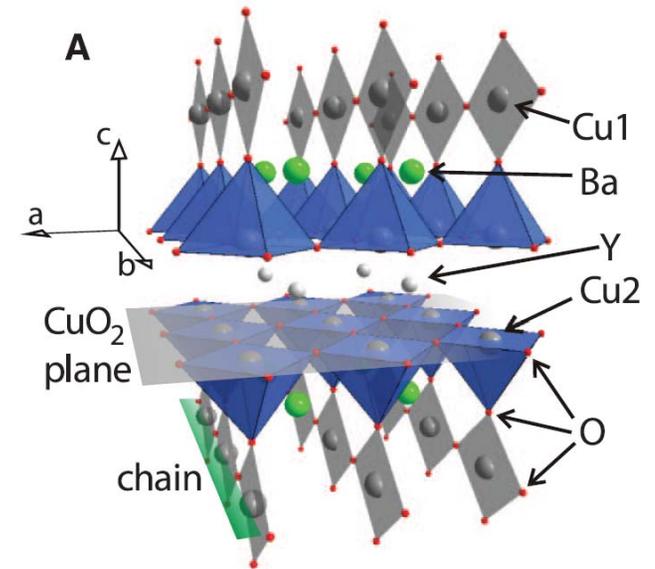


REPORTS

Long-Range Incommensurate Charge Fluctuations in $(Y,Nd)Ba_2Cu_3O_{6+x}$

G. Ghiringhelli,^{1*} M. Le Tacon,² M. Minola,¹ S. Blanco-Canosa,² C. Mazzoli,¹
N. B. Brookes,³ G. M. De Luca,⁴ A. Frano,^{2,5} D. G. Hawthorn,⁶ F. He,⁷ T. Loew,²
M. Moretti Sala,³ D. C. Peets,² M. Salluzzo,⁴ E. Schierle,⁵ R. Sutarto,^{7,8} G. A. Sawatzky,⁸
E. Weschke,⁵ B. Keimer,^{2*} L. Braicovich¹

www.sciencemag.org **SCIENCE** VOL 337 17 AUGUST 2012



Explains mysterious periodicities observed in quantum oscillation experiments as change in Luttinger volume due to folding of the Fermi surface.

Work in Progress: nonlocality?



Contents lists available at SciVerse ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc



Resonant soft X-ray scattering, stripe order, and the electron spectral function in cuprates

Peter Abbamonte^{a,b,*}, Eugene Demler^c, J.C. Séamus Davis^{d,e,f}, Juan-Carlos Campuzano^{g,h}

^a Department of Physics and Frederick Seitz Materials Research Laboratory, University of Illinois, Urbana, IL 61801, USA

^b Advanced Photon Source, Argonne National Laboratory, Argonne, IL 60439, USA

^c Department of Physics, Harvard University, Cambridge, MA 02138, USA

^d Laboratory for Atomic and Solid State Physics, Department of Physics, Cornell University, Ithaca, NY 14853, USA

^e Condensed Matter Physics and Materials Science Department, Brookhaven National Laboratory, Upton, NY 11973, USA

^f School of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews, Fife KY16 9SS, UK

^g Materials Science Division, Argonne National Laboratory, Argonne, IL 60439, USA

^h Department of Physics, University of Illinois, Chicago, IL 60607, USA

In the extreme nonlocal limit:

$$I'_{RSXS}(\mathbf{q}, \omega) \sim \left| \int d\mathbf{r} \int_0^\infty d\omega' \frac{A_{STM}(\mathbf{r}, \omega)}{\omega - \omega' - i\eta} e^{-i\mathbf{q} \cdot \mathbf{r}} \right|^2$$



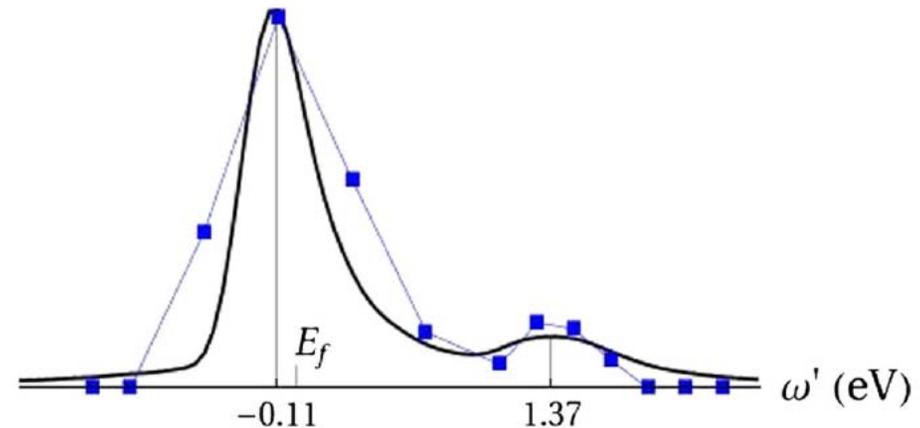
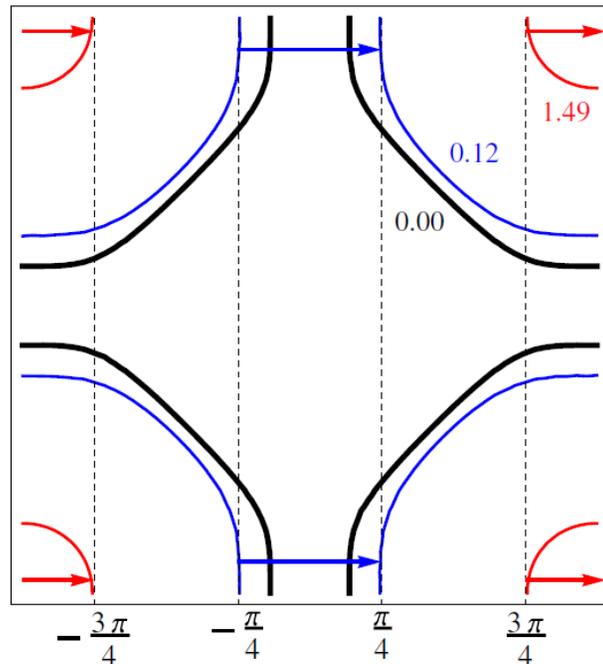
Microscopic Theory of Resonant Soft-X-Ray Scattering in Materials with Charge Order: The Example of Charge Stripes in High-Temperature Cuprate Superconductors

David Benjamin,¹ Dmitry Abanin,¹ Peter Abbamonte,² and Eugene Demler¹

¹*Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA*

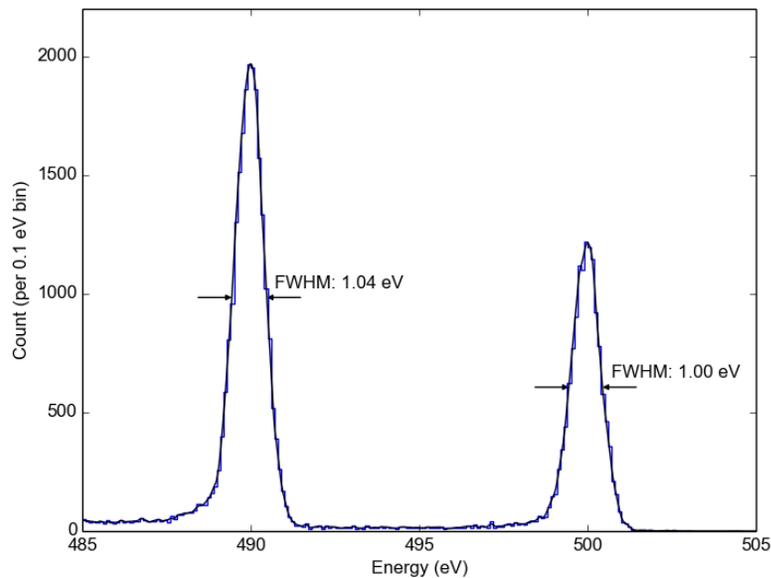
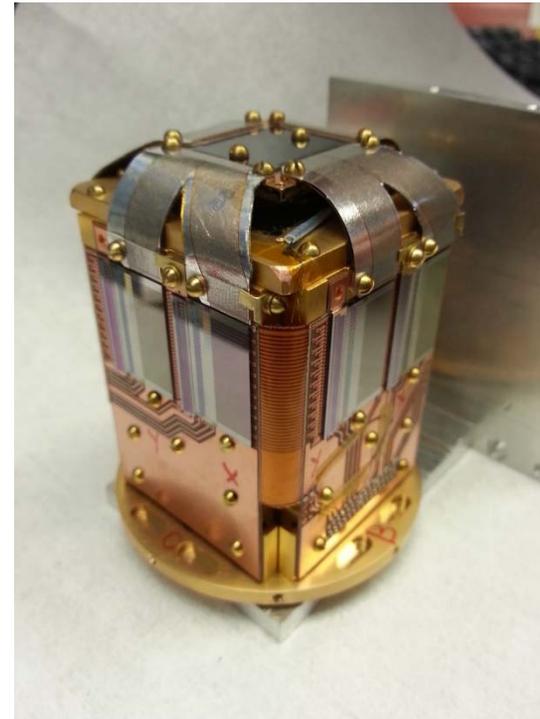
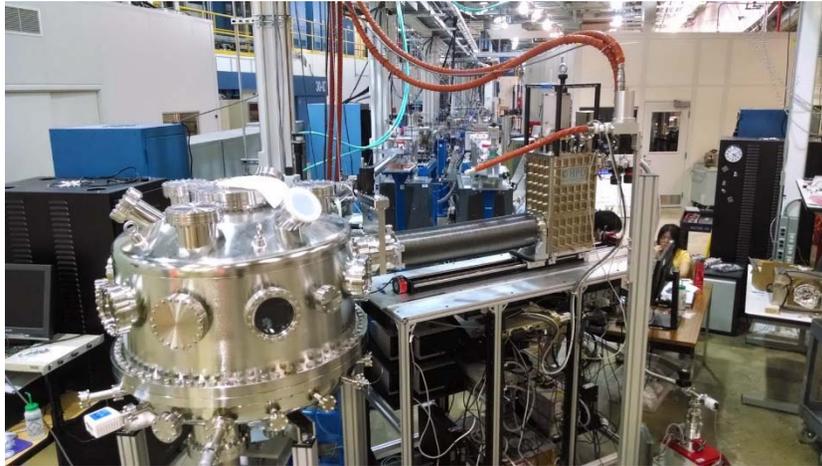
²*Department of Physics and Frederick Seitz Materials Research Laboratory, University of Illinois, Urbana, Illinois 61801, USA*

(Received 14 September 2012; published 26 March 2013)



- Charge amplitude may be small after all. Uh oh!
- If I was wrong all this time, at least I can claim to have helped show it.

Rapid Developments: Better detectors



- 1 eV resolution with no spectrometer
- Eliminate fluorescence background
⇒ study glassy order
- Credit: Joel Ullom, Dan Swetz, Randy Doriese, NIST, Boulder, CO