Catching Nature in the Act: Real-Time Imaging of Quantum Systems with LIQU$i|\rangle$

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GOALS AND OUTLINE

The behavior of quantum systems can be imaged in real-time by measuring dynamical correlation functions. Classically calculating these functions is extremely difficult; however, they can be readily measured on Quantum Computers. My goal is to explicitly calculate the dynamical correlation functions for several ferromagnetic and antiferromagnetic geometries using LIQU$i|\rangle$'s Hamiltonian Mode simulator and show their agreement with the physics of the Heisenberg model.

I will begin with an overview of what these correlation functions mean, especially in the context of quantum spin systems, and how they can be calculated on a quantum computer. Then, I will move on to their circuit's time complexity, susceptibility to error, and the challenges I needed to overcome in order to implement the circuit in LIQU$i|\rangle$. I will conclude with the results of the calculation and the physics they contain.

I. INTRODUCTION

Among the general public, the excitement surrounding quantum computing has centered around Shor's prime factorization algorithm and its implications for certain three-letter agencies. However, quantum computing was originally envisioned as a much more powerful method of simulating complex many-body quantum physics compared to classical computers[1]. Simulating these complex systems means an unprecedented understanding of the chemistry of large molecules, the physics of solid-state materials, and even deep theoretical concepts in Quantum Field Theory[1][2]. Additionally, there are also stunning consequences for those who do not care for the study of nature; the net worth of the different industries that would radically change with access to even a 50-100 qubit computer is somewhere in the trillions of dollars[2].

The question then arises, what properties does one need to know to 'understand' a quantum system? Some of the most important quantities of interest are the energy and symmetries of the eigenstates, phase transitions, physical properties (e.g. heat capacity), degeneracies of the ground state, transport properties, and topological features. Despite addressing widely different aspects of the underlying physics, all of these quantities can be studied through taking expectation values (i.e. quantities such as $\langle\psi|\mathbf{A}|\psi\rangle$) of various operators on the eigenstates. In fact, some of these quantities, like order parameters of a phase transition, are expectation values by their very definition. For example, determining how much power is lost in a transistor boils down to taking expectation values of current operators with the Kubo formula[3].

In addition to being an important theoretical tool for understanding physical systems, expectation values are how we interact with nature. Experimental techniques including Angle-resolved Photoemission Spectroscopy, Neutron Scattering, X-ray Scattering, Scanning Tunneling Microscopy, Raman and Infrared Spectroscopy, etc. all measure different expectation values. This means that using quantum computers to calculate expectation values is not only valuable for improving conceptual understanding, but also produces verifiable predictions that can be checked by many different types of experiments.

One may naïvely wonder, if quantum computers have direct access to the wavefunctions, why not record the wavefunctions themselves instead of measuring expectation values? Surprisingly, measuring complete wavefunctions, something known as a tomography, is neither efficient nor useful. In fact, tomographies are so incredibly inefficient that a quantum computer provides almost no improvement over classical computers in calculating them[2]. Ignoring the efficiency problem, even if the full wavefunctions were available, they would contain roughly 2^{N+1} numbers for N qubits, making them so large that one would end up taking expectation values just to understand them! Thus, to translate quantum systems into something we can understand, there is no way around taking expectation values.

I.1. Correlation Functions

Typically, one does not just take a single expectation value of a quantum state and call it a day. Instead, it is more useful to take expectation values *as a function* of some physical parameter. Returning to the example of heat dissipation in transistor, determining how much heat is dissipated *as a function* of the transistor switching speed is extremely important for the semiconductor industry. Technically speaking, parameter dependent expectation values are generally referred to as correlation functions. In the case that one of these parameters is time, the function is termed a *dynamical* correlation function.

In this project, I studied spin systems, so naturally I needed to find the appropriate correlation functions to calculate. Within spin systems, the first property to determine is whether the system is ferromagnetic or not. Ferromagnetism can be easily determined by adding up the expectation value of the local vector spin at each po-

sition in space, as shown in Equation 1.

$$\langle \vec{\mathbf{S}} \rangle = \sum_{\vec{\mathbf{r}}} \langle \vec{\mathbf{S}}(\vec{\mathbf{r}}) \rangle = \sum_{\vec{\mathbf{r}}} \langle \psi | \vec{\mathbf{S}}(\vec{\mathbf{r}}) | \psi \rangle \tag{1}$$

Here $\vec{\mathbf{S}}(\vec{\mathbf{r}})$ is the spin vector operator acting at position $\vec{\mathbf{r}}$, and $\langle \vec{\mathbf{S}}(\vec{\mathbf{r}}) \rangle$ is its correlation function. One can understand $\langle \vec{\mathbf{S}}(\vec{\mathbf{r}}) \rangle$ as a map of how the spins are aligned locally in space, and the sum as giving the total magnetization. A finite value of this sum would indicate the ground state has a net magnetic moment and is ferromagnetic (or ferrimagnetic).

One can also look at the dynamical correlation function $\langle \vec{\mathbf{S}}(\vec{\mathbf{r}}, t) \rangle$ to obtain a movie of how, if at all, the spins in the system are changing direction in time.

Multiple operator correlation functions, such as $\langle \vec{\mathbf{S}}(\vec{\mathbf{r}'}, t') \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}, t) \rangle$, go a step beyond their single operator versions and contain information about how different parts of the system interact with each other. As an example, for those interested in developing electronics based on spin, knowing how the rotation of a spin at a given place and time $(\vec{\mathbf{r}}, t)$ affects another spin at some other place and time $(\vec{\mathbf{r}'}, t')$ is vital for determining when spin currents can flow easily, and when they would just dissipate into heat.

To get an understanding of what a movie of a multiple operator dynamical correlation function would look like, consider the analogy of throwing a small pebble into a still pond at some position $\vec{\mathbf{r}}$ at time t_0. The pebble would make a splash, and waves of water will cause the water level at other positions $\vec{\mathbf{r}'}$ to oscillate. After long enough time however, the water level will settle back down and the waves will have dissipated into random water motion. One can imagine how the results would be different for gelatin, where not only would the pebble cause waves to form like the case of water, but the gelatin as a whole would vibrate, which water does not. Though the analogy is crude, similar behavior occurs for ferromagnets and antiferromagnets, because antiferromagnets have some excitations that simply do not exist for ferromagnets. By measuring a multiple operator dynamical correlation function, you can determine just what the system's excitations are, and how they affect the interactions between particles in the system.

In this project, I mainly studied a dynamical correlation function that embodies the 'pebble' concept for spin systems, and is defined in Equation 2.

$$\langle S_{+-}(i, j, t, t') \rangle \equiv \langle S_{+}(j, t') S_{-}(i, t) \rangle \tag{2}$$

Here, $S_{\pm} = S_x \pm i S_y$ is the spin raising(lowering) operator. This correlation function is known as the transverse correlation function. Though its appearance is daunting, the transverse correlation function answers a rather simple question: if a spin-up state $|\uparrow\rangle$ is lowered to a spin-down state $|\downarrow\rangle$ at position and time (i, t), what is the probability that the $|\downarrow\rangle$ state propagates to (j, t')? For spins that are static (possibly because of an external field), there will be no probability for propagation. On the other hand, for strongly-coupled systems, propagation can occur in many different channels at once.

II. MANY-BODY SPIN SYSTEMS

Before continuing, it is instructive to review some of the physics behind the ferromagnetic and antiferromagnetic systems I studied in this project.

Classically, magnetic systems, such as iron, are imagined as lattices of small bar magnets pointing in the same direction, each interacting with the other through a dipole-dipole force. Astonishingly, the origin of magnetism at the quantum level is not from electromagnetism, but instead comes from spin statistics! The core idea is simple: in order to avoid being in the same state, electrons in a material are forced to align their spins and/or angular momentum in parallel, antiparallel, or somewhere in between the two. This forced (anti-)alignment is why magnetism is an important aspect of strongly-coupled quantum systems.

The Heisenberg model shown in equation 3 is one of the simplest models of quantum magnetism, yet for different geometries and values of the parameter J it contains the exotic spin states that could be used as topological quantum computers or to explain high-temperature superconductivity[4].

$$H = \frac{J}{2} \sum_{i,j} \vec{\mathbf{S}}_i \cdot \vec{\mathbf{S}}_j \tag{3}$$

A useful property of the Heisenberg model for the purpose of this paper is its time invariance. This property means one time index is redundant for any two or more operator correlation function, as shown in Equation 4. The geometries I used also had translation invariance, which is to say that every site couples to the other in the exact same way. Translation invariance acts very similar to time-invariance, and makes one position index redundant. So instead of calculating $\langle S_{+-}(i, j, t, t') \rangle$, only $\langle S_{+-}(i = 0, j, t = 0, t' - t) \rangle \cong \langle S_{+-}(j, t') \rangle$ is needed.

$$\langle A(t') B(t) \rangle = \langle A(t' - t) B(0) \rangle \equiv \langle A(\tau) B(0) \rangle \tag{4}$$

There are two distinct regimes of the Heisenberg model depending on the value of J. For $J < 0$ spins will try to align, resulting in a ferromagnet, and for $J > 0$ spins will try to anti-align, resulting in an antiferromagnet.

II.1. Ferromagnetic Heisenberg Systems

The ferromagnetic Heisenberg model in all geometries has the classically expected ground state of all spins aligned in the same direction (e.g. $|\uparrow\uparrow \cdots \uparrow\rangle$). The direction of the spins is usually called the longitudinal direction, and any perpendicular direction is considered to be transverse.

In the ferromagnetic case, the longitudinal correlation function $\langle S_{zz}(i,t)\rangle$ is simply a constant. This is simply the statement that the spin at any site is aligned with the spin of any other site forever in time. On the other hand, the transverse correlation function $\langle S_{+-}(i,t)\rangle$ is far from trivial.

To get an intuitive feel for what to expect, recall that the operator $S_\pm$ act to create a $|\uparrow\rangle$ and $|\downarrow\rangle$ state respectively. This means the system goes from $|\uparrow\uparrow\cdot\cdot\uparrow\rangle$ to $|\uparrow\downarrow\cdot\cdot\uparrow\rangle$. This new state is no longer an eigenstate of the Heisenberg Hamiltonian. Having just one spin misaligned is unstable; so the system spreads the localized spin misalignment over all the spins, giving each spin a much smaller misalignment. The spreading of a single misalignment is carried by spin waves excitations known as magnons, and they underpin the field of magnonics. Thus, we should expect to see the initial spin flip create waves that move in all directions at a finite speed related to the coupling constant J[4].

II.2. Antiferromagnetic Heisenberg Systems

While the ground state of the ferromagnetic regime is rather simple, the Heisenberg antiferromagnet is highly dependent on geometry. Classically, one would expect that the ground state would be the Neél state with anti-aligning spins $|\uparrow\downarrow\uparrow\downarrow\cdot\cdot\uparrow\downarrow\rangle$. However, this state is not an eigenstate, and for this reason the Neél state is said to be altered by 'quantum fluctuations'.

Because the ground states depend on geometry, the antiferromagnetic correlation functions do as well. In general, the local spin $\langle\vec{\mathbf{S}}(\vec{\mathbf{r}})$ will be aligned between all sites, and will alternate in sign like classical Neél order. An important difference from Neél order however, is that quantum fluctuations will decrease the magnitude of $\langle\vec{\mathbf{S}}(\vec{\mathbf{r}})\rangle$ and cause it to be time-dependent.

The transverse correlation functions also takes a different form than the ferromagnetic case. While magnons are still present like in the ferromagnetic regime, so-called 'spinons' also exist. These spinons are best illustrated by example. Starting from the Neél state $|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle$, flipping a single spin causes three spins to be aligned in a row $|\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\rangle$. This state is not stable however, and these spins can move to give $|\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\rangle$. These spinons are effectively domain walls between two Neél states, and can propagate in the system much like conventional particles do in free-space.

III. METHODS

III.1. Measuring Correlation Functions

By design, quantum computers only allow for two types of operations: unitary rotations, and measurements. In effect, the only expectation value that can be obtained is that of $\langle\sigma_z\rangle$, because it is simply the difference in the number of times $|\downarrow\rangle$ and $|\uparrow\rangle$ are measured.

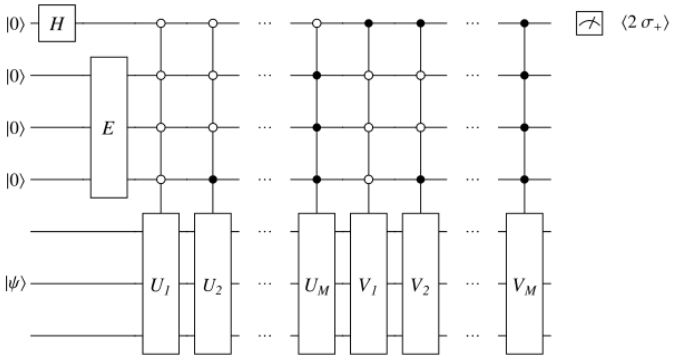

FIG. 1. General circuit for measuring linear sums of unitary operators. The image is taken from Collins[2] and originally from Somma[6].

This raises the obvious question of how generic expectation values can be evaluated, considering that most operators are non-unitary. This limitation may seem like an insurmountable barrier; however, there is mathematical theorem stating that any complex matrix can be written as a linear sum of at most two unitary matricies[5]. So the problem of taking non-unitary expectation values is reduced to unitary expectation values, as the results could be scaled and added with impunity after the fact.

The most general statement of the problem is:

"Is it possible to measure the real and imaginary parts of the expectation value of a linear sum of 2^N unitary matrices $\sum_{i=0}^{2^N} a_i U_i^\dagger V_i$ for $\sum_{i=0}^{2^N} |a_i|^2 = 1$?"

This problem can indeed be solved[2,6,7], and requires N ancilla qubits set up in the circuit shown in Figure 1. The circuit essentially works by first rotating the ancilla qubits from $|00\cdots0\rangle$ to $\sum_i^{2^{N-1}} a_i|i\rangle|+\rangle$, denoted by the gate E. The circuit then applies N-qubit controlled unitary operators for each $a_i|i\rangle$. Finally, measuring $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ gives the real and imaginary parts of the sum. This last measurement can be done by rotating the first qubit into the x and y bases and measuring $\langle\sigma_z\rangle$. As an interesting sidenote, in the process of actually implementing this circuit, I found that the textbook reference[2] on this circuit not only skipped the application of the E gate, but also added two erroneous steps at the end that can only be understood as typos.

An immediate problem with the scheme in Figure 1 is that it collapses the system wavefunction into an unknown state upon measurement. If the wavefunction was obtained with an annealing procedure, the whole annealing process would need to be repeated for each measurement. One alternative is to use a combination of phase estimation and the Hellmann-Feynman theorem to get around wavefunction-collapse. The Hellmann-Feynman theorem is a very elementary result which states that for a Hamiltonian of the form $H(\lambda) = H_0 + \lambda O$, Equation 5 holds[8].

$$\frac{d\langle E\rangle}{d\lambda}\Big|_{\lambda=0} = \langle O\rangle \tag{5}$$

Using this theorem, instead of measuring the operator

directly, one can use phase estimation on $U(\lambda)$ to obtain $E(\lambda)$ and numerically compute the derivative. Phase estimation not only measures the energy of the state, but, for enough ancilla qubits, collapses the wavefunction to the exact eigenstate corresponding to that energy. Thus, phase estimation allows for non-destructive measurement. As an aside, because λO needs to be Hermitian, in general operators will need to separated into Hermitian and anti-Hermitian parts to get a full complex expectation value.

There is a difficulty in using this approach, however, due of its unsuitability for dynamical correlation functions. This difficulty arises because adding terms to the Hamiltonian of the form $U^\dagger(t)OU(t)$, where $U(t) = \exp(-\frac{i}{\hbar}H_0 t)$, is highly nontrivial. In general, if one knew how to compute $U^\dagger(t)OU(t)$, there would be no need to measure its expectation value in the first place.

III.2. Time Complexity

Before attempting to use either method for measuring correlation functions, the time-complexity must be assessed. For the scheme in Figure 1, it is useful to break the problem into two pieces.

The first piece is the number of repetitions needed to get measurements with absolute precision $\pm\sigma$. For a single ancilla measurement, the results will have a binomial distribution, requiring $O(\frac{1}{\sigma^2})$ iterations for the desired precision.

The second is the number of unitary time steps needed. If the correlation function is to be evaluated at T time points and for n values of its parameter, $O(n(1+2+...+T)) = O(nT^2)$ time steps will be needed for each operator. So for a m-operator dynamical correlation function, $O(n^m T^{2m})$ time steps are needed. If the system has a time-independent and translationally-invariant Hamiltonian (e.g. the Heisenberg model on a ring), the number of time steps drops to $O(n^{m-1}T^{2m-1})$.

The total number of time evolutions then scales as $O(\frac{K}{\sigma^2}n^m T^{2m})$, where K is the constant number of time-steps needed to anneal the system to the proper ground state in the first place. For a fixed precision and number of operators, the circuit scales polynomially in the unitary time steps it needs. The exponential scaling in the number of operators may seem alarming, but it is to be expected, as the sheer number of parameters in a multi-operator correlation functions also scales exponentially.

If however, there is an efficient method for calculating and applying U^{2^n}, the situation changes. Assuming these matricies can be calculated for a 2^n such that $2^{n+1}-1 \geqslant T$, the number of time steps would be $1+2+..+\log_2(T)$ or $O(\log_2^2(T))$. This can be seen by simply writing T in binary and applying the appropriate U^{2^s} for each digit. If this speedup is possible, then the time complexity will be $O(\frac{K}{\sigma^2}n^m \log_2^{2m}(T))$.

On the other hand, the phase estimation and Hellmann-Feynman theorem scheme scales rather differently. To resolve an energy difference on the order of λ, the number of qubits required for phase estimation is roughly $\log_2(\frac{1}{\lambda})$. If $U^{2^s}(\lambda)$ can be efficiently computed beforehand and has the same cost as $U(\lambda)$, $O(\log_2(\frac{1}{\lambda}))$ time evolutions are needed. In the general case of m operators at n points the complexity becomes $O(n^m \log_2(\frac{1}{\lambda}))$. If there is no efficient method of obtaining $U^{2^s}(\lambda)$, then $O(\frac{n^m}{\lambda})$ unitary time steps are needed. In either case, there is a significant speedup in using the phase estimation method. However the downside for small system sizes is the number of additional qubits needed, which could be around ten for reasonable accuracy. Combining this downside with the unsuitability for dynamic correlation functions, the method described in Figure 1 had to be used for actual measurements in my project because the LIQU$i|\rangle$ simulation is extremely slow as the number of qubits grow.

III.3. Error Susceptibility

For either of the aforementioned methods, the main error source will come from controlled and uncontrolled unitary time steps. In particular the circuit would be most susceptible to error when measuring correlation functions at long times for the first method, or with too high precision for the second method.

In addition, measuring correlation functions is intrinsically very sensitive to depolarizing noise, as adding a single X, Y, Z gate for any qubit will effectively change the operators involved in the expectation value. For example, in the case of $\langle S_{xx}(i=0,j,t)\rangle$, an additional X gate at the $i=0$ position would completely undo the first S_x operation and cause the expectation value to be identically zero for a ferromagnet. The net effect of these errors would generically be to bias expectation values towards zero.

For translationally invariant systems, a more stable computation would be the sum of $\langle S_{xx}(i=0,j,t)\rangle$ over all equivalent sites $\sum_i^N \langle S_{xx}(i,j,t)\rangle = N\langle S_{xx}(i=0,j,t)\rangle$. By computing a sum, polarizing errors may be circumvented provided much less than N occur. In this sense, the second method is less fatally susceptible to polarizing errors because phase estimation can bring the system back to the correct eigenstate, but there is the additional worry that the state will move to another energy level if errors are allowed to accumulate.

In light of the weakness of these two methods to errors, error correction is absolutely necessary to reliably measure correlation functions with a quantum computer.

IV. IMPLEMENTATION IN LIQU$i|\rangle$

The implement a generic correlation function calculator in LIQU$i|\rangle$, I focused on two aspects: building utilities to create arbitrary lattice Hamiltonians, and creating the expectation value measurement circuits. The utilities I needed were a matrix exponentiator to allow the `Spin` class to take in arbitrary terms, a geometry class

that would handle the qubit coupling configurations, and an integrated noise extension to the `Spin` class. The measurement circuits required a helper function to decompose complex matricies into linear sums of unitary matricies, building the circuit in Figure 1, and building the circuit for the Hellmann-Feynman method. In effect, because LIQU$i|\rangle$'s `Spin` class is able to handle matrix optimization, trotterization, and annealing schedules, it handled all of the heavy lifting of doing the actual quantum computation.

IV.1. Running Arbitrary Hamiltonians

The `Spin` class is fundamentally designed to handle arbitrary Hamiltonians through the `SpinTerms` class. However, `SpinTerms` requires a pre-existing exponentiation of each matrix and cannot accept Hermitian matricies on their own. Matrix exponentiation is very simply accomplished by diagonalizing the relevant matrix, something that is not very expensive for the small local terms that are used in most Hamiltonians. Thus, I implemented a generic rotation matrix by diagonalizing the Hamiltonian term with .NET's Numerics and caching the result.

Apart from the matricies of arbitrary Hamiltonian terms, deciding what qubits they couple is equally important. To avoid having to keep track of qubit IDs, I created a geometry class to handle the qubit coordination. This class simplifies moving a given Hamiltonian between geometries, adding terms beyond nearest-neighbor coupling, breaking/enforcing translation symmetry, etc.

Finally, because the `Spin` class does not have built-in Noise functionality, I added simple methods to do so. While I was able to implement depolarizing error, I was unable to configure the amplitude dampening with the unitary evolution. In principle, error correction could be added as well, but it currently would have limited functionality because of the large number of physical qubits needed for each logical qubit.

IV.2. Correlation Function Circuits

To make the circuit in Figure 1 handle the expectation value of any complex matrix, a four-term unitary decomposition method was used[5]. This method requires computing the singular-value decomposition of the original matrix along with diagonalizing its Hermitian and anti-Hermitian parts. Again, these matrix operations were implemented with the .NET Numerics. Although this procedure is generically very expensive, the operators involved in expectation values usually act on only a few qubits, so their sizes are limited, unlike the full Hamiltonian.

Because LIQU$i|\rangle$ is a fundamentally classical simulator, the circuit in Figure 1 can be implemented in $O(n^m T^m)$ time instead of $O(\frac{K}{\sigma^2} n^m T^{2m})$. This speedup is possible because the `Ket.Probs` method allows one to obtain the result of a measurement perfectly without performing it. So instead of the measuring and collapsing the system, I called `Ket.Probs` and then applied the adjoint of all the applied gates to undo them and move to the next the timestep. Actually implementing the measurements would require much more computational time and only introduces uninteresting statistical error bars. Calculating $\langle S_{xx}(i,j,t)\rangle$ in this semi-classical way for fixed i on a 17 qubit ferromagnetic chain with 3500 time steps took one hour running on an i5-Skylake processor. Sample input and output for the scripts is available in my Dropbox folder here.

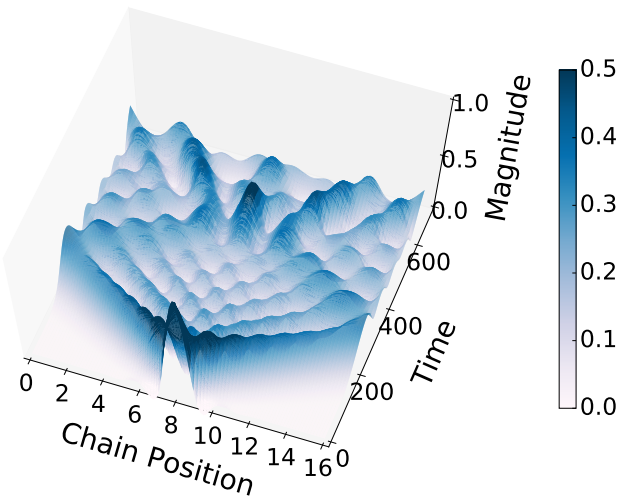

FIG. 2. Surface plot of the absolute value of $\langle S_{+-}(i,j,t)\rangle$ for a ferromagnetic 17 qubit ring and $i = 8$. The initial disturbance propagates away like a coherent wave, as expected for magnons.

Implementing the phase estimation method was also rather simple. The unitary matricies $U(\lambda, O)$ were created by adding $e^{i\lambda\theta O}$ to the annealing schedule of the `Spin` object used. Measurement could be made exact by using the `Spin.EnergyExpectation()` method; however, because the Hellmann-Feynman method has trouble working for dynamical and multi-operator correlation functions, I did not use it in any simulations.

V. RESULTS

Using the correlation function calculator circuit, I obtained the dynamical local spin $\langle \vec{\mathbf{S}}(i,t)\rangle$, transverse correlation function $\langle S_{+-}(i,t)\rangle$, and longitudinal correlation function $\langle S_z(j,t)S_z(i,0)\rangle$ for ferromagnetic and antiferromagnetic rings, square lattices, and hexagonal lattices. All of these datasets are available at my Dropbox folder[9]. For the sake of brevity however, I will only discuss the ring geometry and mention the square lattice case in passing. Because the Heisenberg model, as shown in Equation 3 has only one energy scale, only the sign of J matters, so all datasets were taken with $J = \pm 0.01$, $\hbar = 1$, and scaled so $S_i = \sigma_i$.

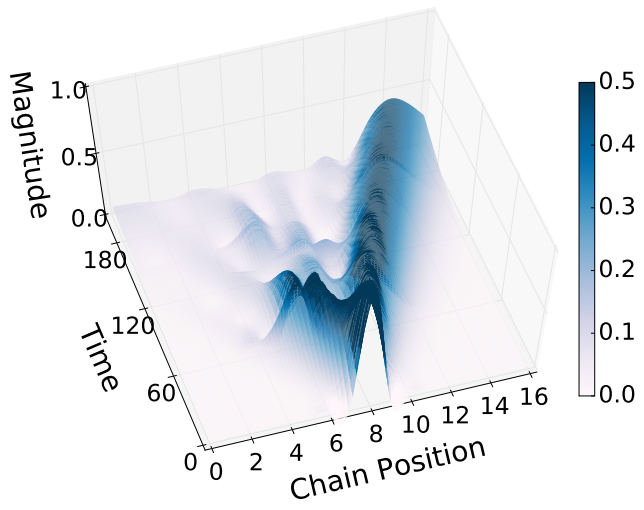

FIG. 3. This plot has the same data as Figure 2, but with three depolarizing noise events. The first event at $t \approx 60$ is enough to spoil the correlation function.

V.1. Ferromagnetic Results

As mentioned before, the ferromagnetic ground state restricts the local spin and longitudinal correlation function to be constants, so all the interesting behavior appears in the transverse correlation function.

Figure 2 shows a interpolated surface plot of the transverse correlation function for 17 qubits aligned in a ring. The spin flip away from $i = 8$ visibly propagates outwards in both directions and returns because the ring is periodic, precisely what the was expected for ferromagnetic magnons. An alternative way to understand this is that because the initial disturbance is spatially localized, it excites magnons in all directions and momentum. To explore the effects of noise, I recalculated $\langle S_{+-}(i,j,t)\rangle$ with depolarizing errors in Figure 3. The original spin flip propagates at the very beginning; however, just a single depolarizing event around $t = 60$ completely destroys the measured correlation function, which is clear proof of how sensitive the correlation function is to depolarization.

V.2. Antiferromagnetic Results

As stated in section II.2, the ground state of the Heisenberg antiferromagnet is non-classical, so I annealed from the Neél ground state of the Ising antiferromagnet over 1000 time steps. To get a clearer look at the spin of the Heisenberg ground state, I created a video of the local spin $\langle \vec{\mathbf{S}}(i,t)\rangle = \langle S_z(i,t)\rangle \hat{\mathbf{z}}$ for a 4-by-4 antiferromagnetic square lattice that is available here.

The transverse correlation function $\langle S_{+-}(i,j,t)\rangle$is calculated in Figure 4 for a 16 qubit ring with $i = 0$. While spin waves still propagate like in the ferromagnetic case, localized spinons also form from the initial spin flip and

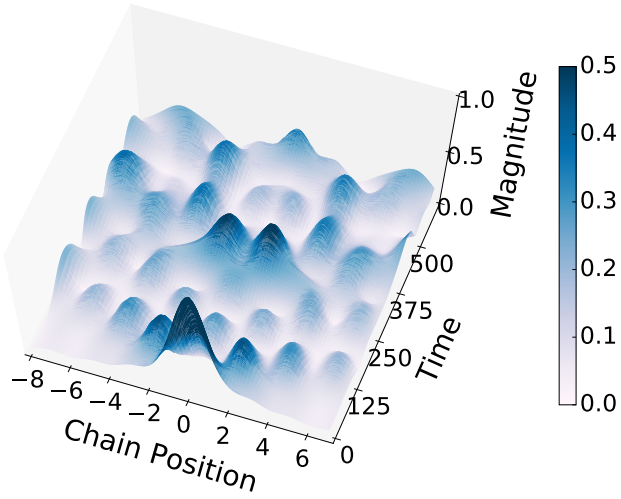

FIG. 4. Surface plot of the absolute value of $\langle S_{+-}(i = 8, j, t)\rangle$ for an antiferromagnetic 17 qubit ring. The disturbances are also much more localized due to spinon modes existing in addition to the wavelike magnons.

are very distinct features compared to the ferromagnetic case, confirming what was expected from section II.2.

VI. CONCLUSIONS

Complex quantum systems are at the bleeding edge of scientific research because of their fundamental importance and financial implications. With LIQU$i|\rangle$'s quantum simulator, I was able to record quantum spin systems in action by measuring their dynamical correlation functions. Along the way, I found errors in the measurement circuit of a well-known textbook, added new classes and functions to LIQU$i|\rangle$ to allow for more universal Hamiltonian simulation, and analyzed the time complexity and error-proneness of the mainstream method for calculating dynamical correlation functions on quantum computers.

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